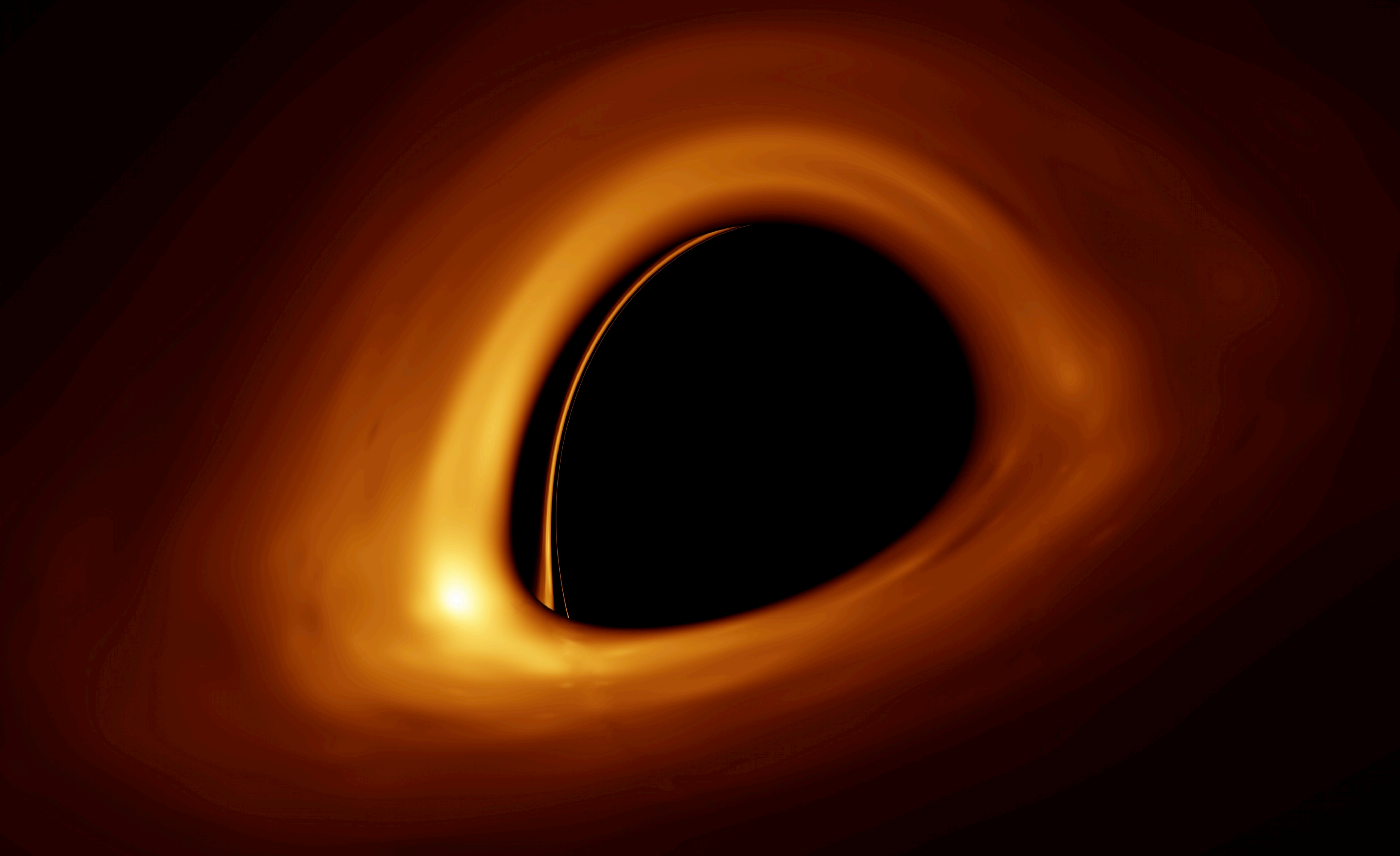
Black Holes and Magnetic Fields

ESO/OPTICON/IAU Summer School
Brno, 1-11 September 2015

Vladimír Karas

Astronomical Institue, Academy of Sciences of the Czech Republic

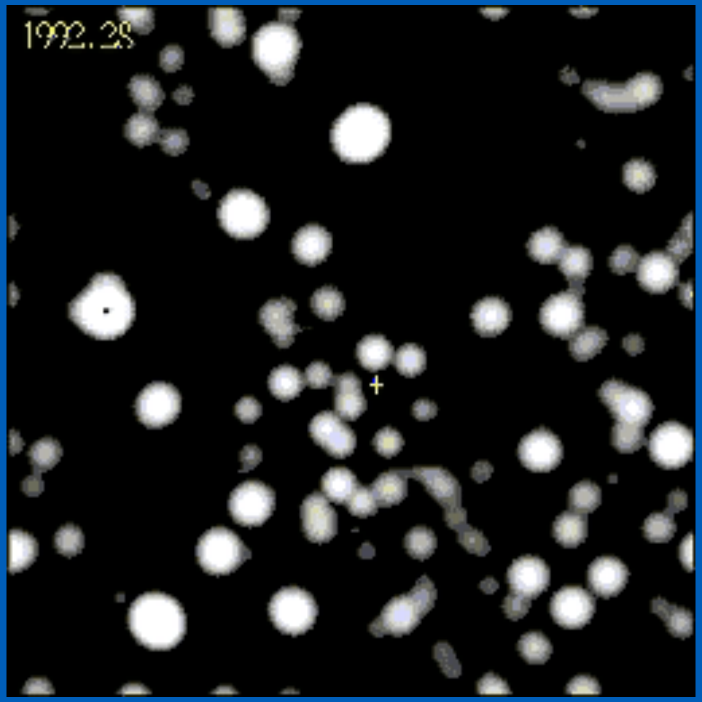
Black Hole Shadow



(no magnetic field)

Bursa et al. (2007)

Center of the Milky Way

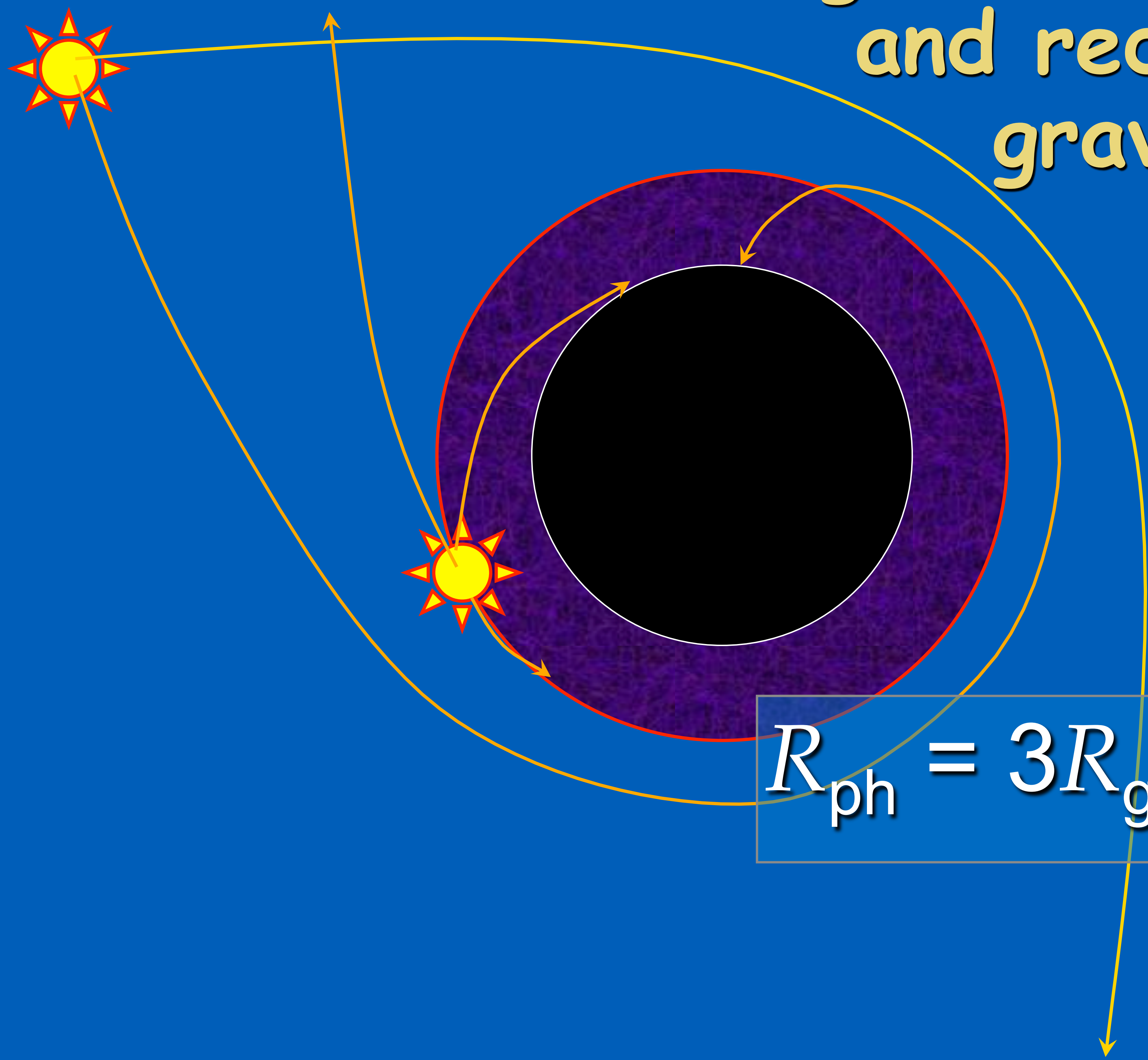


Black hole mass:
$4 \times 10^6 M_S$

A dormant galactic nucleus

Genzel et al.,
Eckart et al.
Ghez et al.

Light deflection, focusing and redshift in strong gravitational fields



$$R_{ph} = 3R_g = 3GM/c^2$$

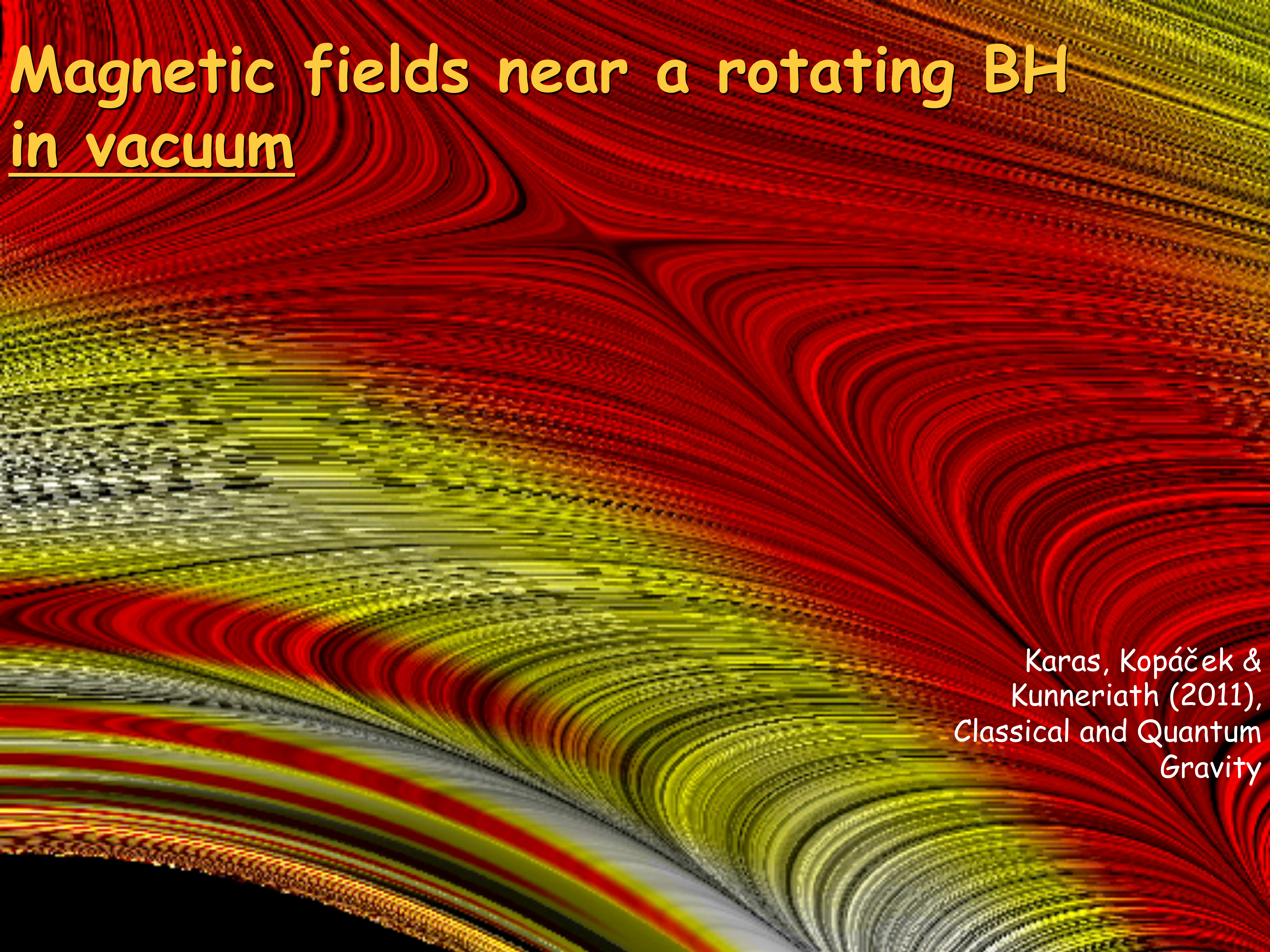
K. Schwarzschild: "Über das Gravitationsfeld eines Masses nach der Einsteinschen Theorie", Sitzungsberichte Königlich Pr. Akad. Wiss., 189 (1916).

Scaling with the mass

$$R_g = GM_{BH}/c^2$$
$$\cong 1.5 \cdot 10^5 \left(M_{BH}/M_S\right) \text{ cm}$$

- **Super-massive** black holes (~ $10^6 - 10^9\, M_S$)
  - In the cores of most galaxies (including the Milky Way)
  - Prime movers of active galaxies
- **Intermediate** holes (~ $10^3\, M_S$)
  - Possibly in collapsed cores of stellar clusters
- **Stellar-mass** black holes
- **Low-mass** black holes – not of current observational interest

Magnetic fields near a rotating BH in vacuum



Karas, Kopáček & Kunneriath (2011), Classical and Quantum Gravity

Some historical remarks

John Michell (1784): Phil. Trans. Roy. Soc. Lond. LXXIV, 35

If there should really exist in nature any bodies whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us … we could have no information from sight; yet if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones …

Pierre S. Laplace (1796): "Exposition du Systēme du Monde"

… the attractive force of a heavenly body could be so large that light could not flow out of it.

(see W. Israel, in "300 Years of Gravitation")

On the maximum mass

S. Chandrasekhar (1931): "The maximum mass of ideal white dwarfs", Astrophysical Journal 74, 81

... there is no cause in the quantum theory that could prevent collapse of a body of the mass M > M_0 in a point …

Limiting mass:

white dwarfs ≈ 1.4 M_S, neutron stars ≈ 2 M_S

S. Eddington (1935): "Relativistic degeneracy", Observatory 58, 37

W. Baade & F. Zwicky (1934): "On supernovae; Cosmic rays from supernovae", Proc. Nat. Acad. Sci. 20, 254

J. R. Oppenheimer & H. Snyder (1939): "On continued gravitational contraction", Phys. Rev. 56, 455

Equation of hydrostatic equilibrium ("TOV")

J.R. Oppenheimer & G. Volkoff: "On massive neutron cores", Phys. Rev. 55, 374 (1939)

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left(\tilde{n} + p\right)\left(m + 4\eth r^3 p\right)}{r\left(r - 2m\right)}$$

On the maximum mass

S. Chandrasekhar (1931): "The maximum mass of ideal white dwarfs", Astrophysical Journal 74, 81

> *... there is no cause in the quantum theory that could prevent collapse of a body of the mass $M > M_0$ in a point …*

Limiting mass:

white dwarfs ≈ 1.4 M_S, neutron stars ≈ 2 M_S

S. Eddington (1935): "Relativistic degeneracy", Observatory 58, 37

W. Baade & F. Zwicky (1934): "On supernovae; Cosmic rays from supernovae", Proc. Nat. Acad. Sci. 20, 254

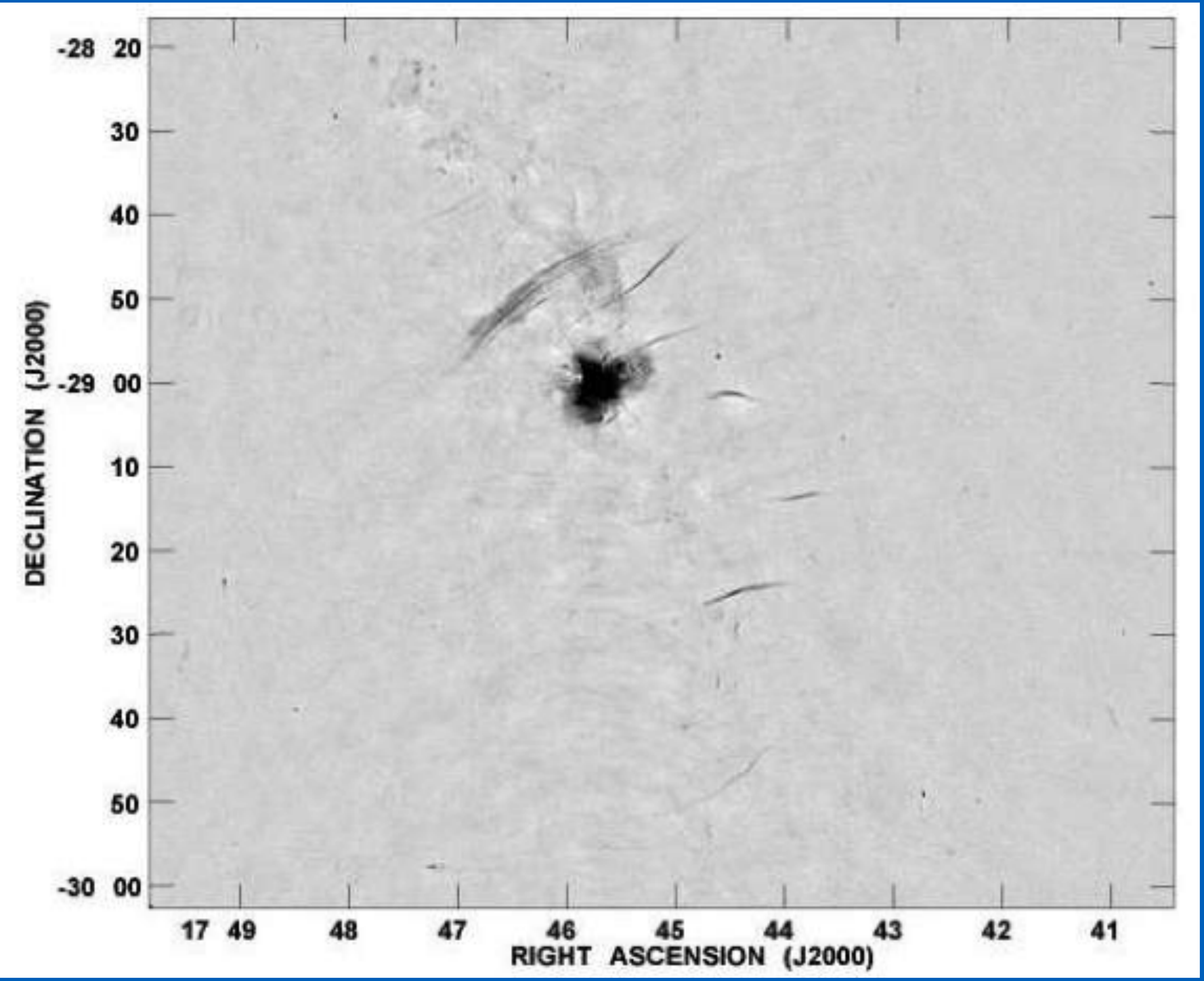
J. R. Oppenheimer & H. Snyder (1939): "On continued gravitational contraction", Phys. Rev. 56, 455

Equation of hydrostatic equilibrium ("TOV")

J.R. Oppenheimer & G. Volkoff: "On massive neutron cores", Phys. Rev. 55, 374 (1939)

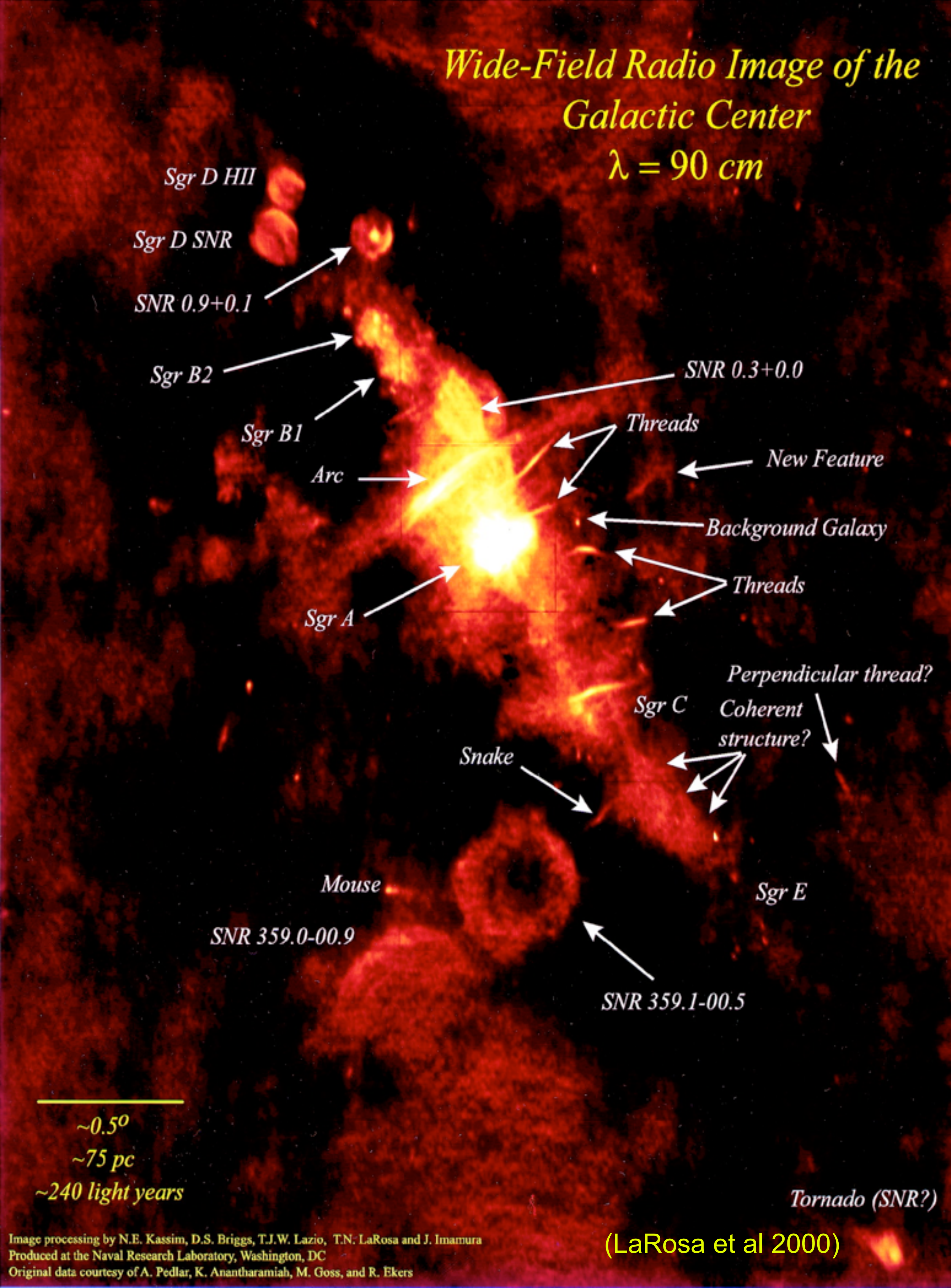
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(\tilde{n} + p)(m + 4\eth r^3 p)}{r(r - 2m)}$$

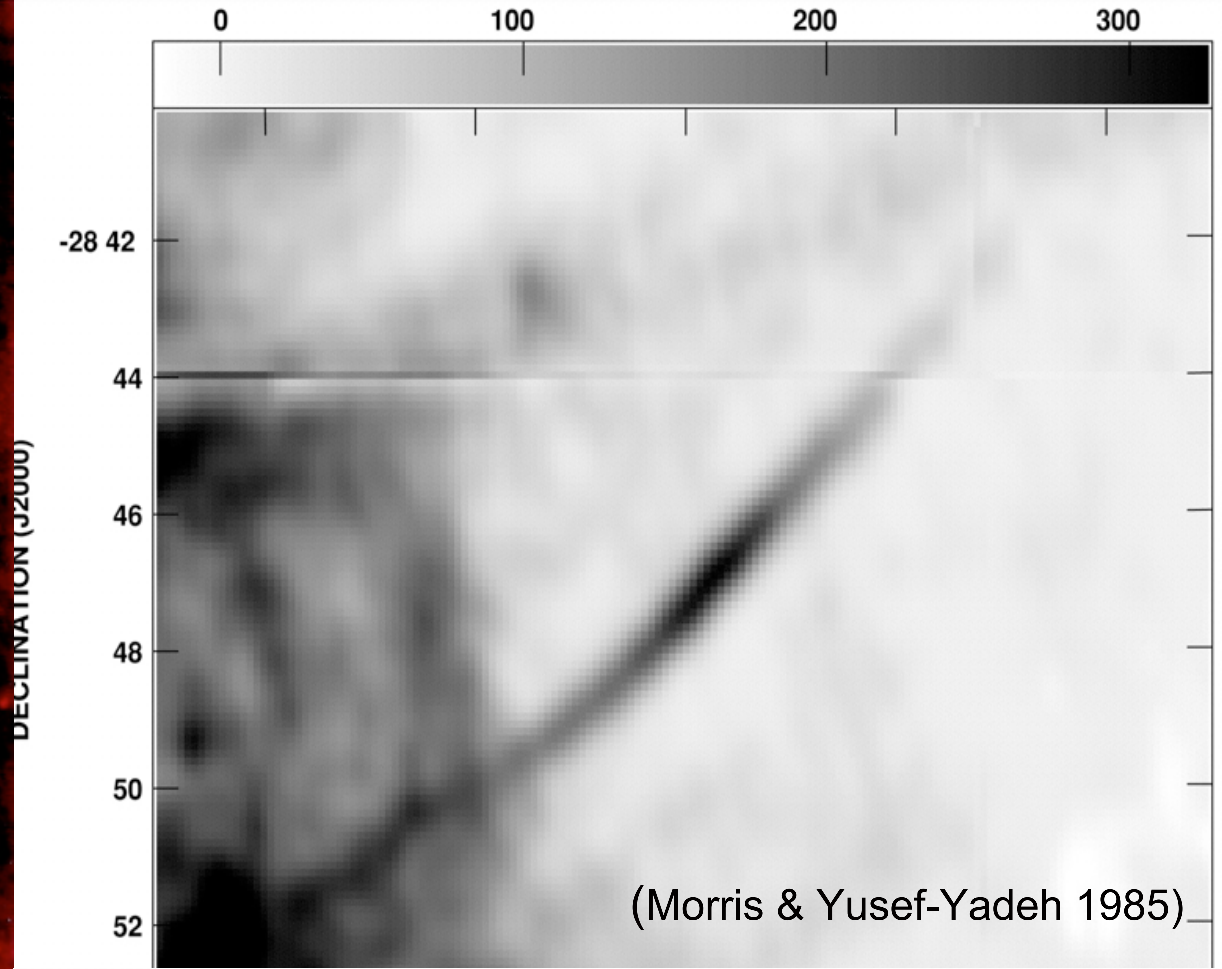
Motivation #1:
Non-thermal filaments in Galactic Center

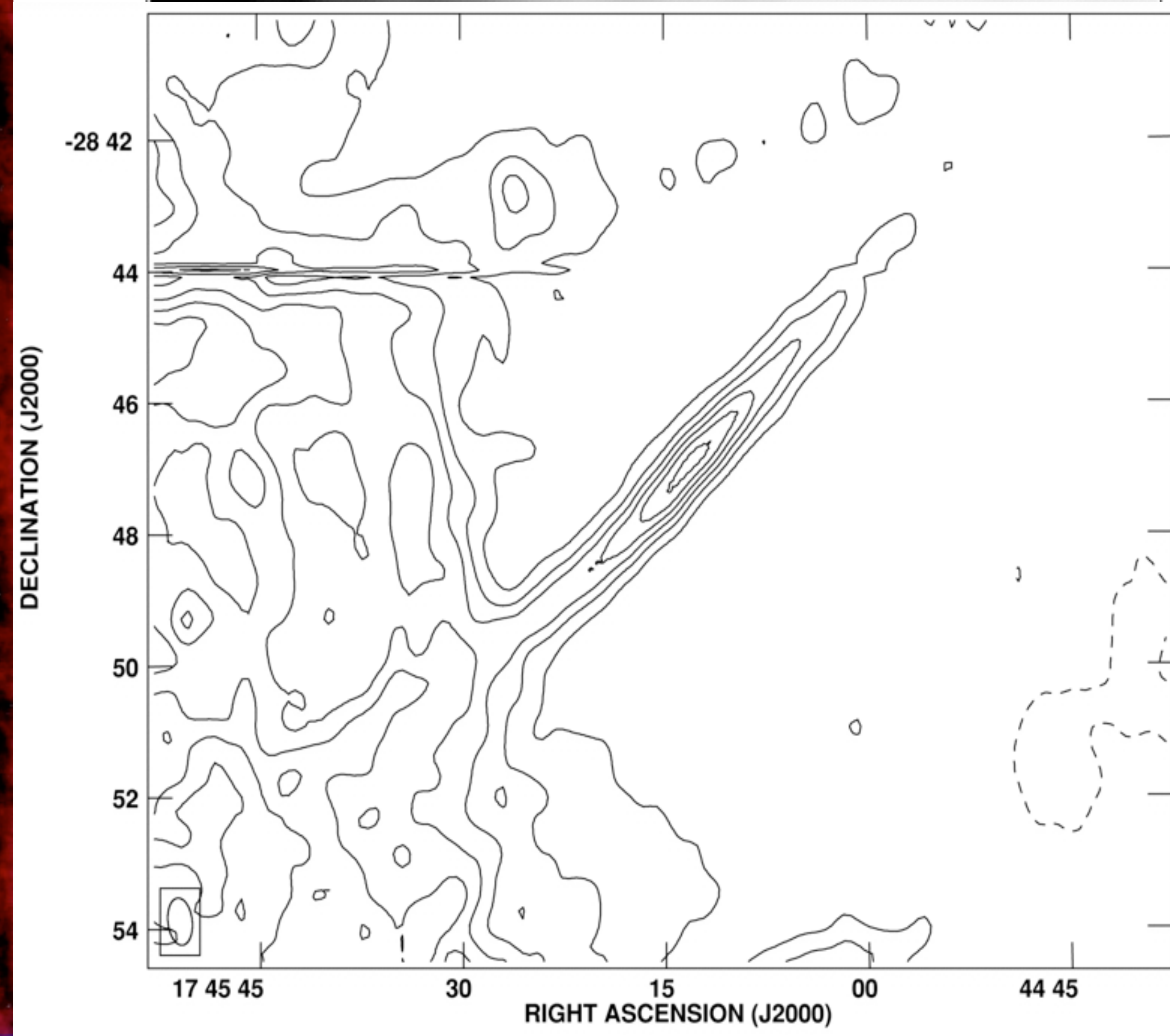


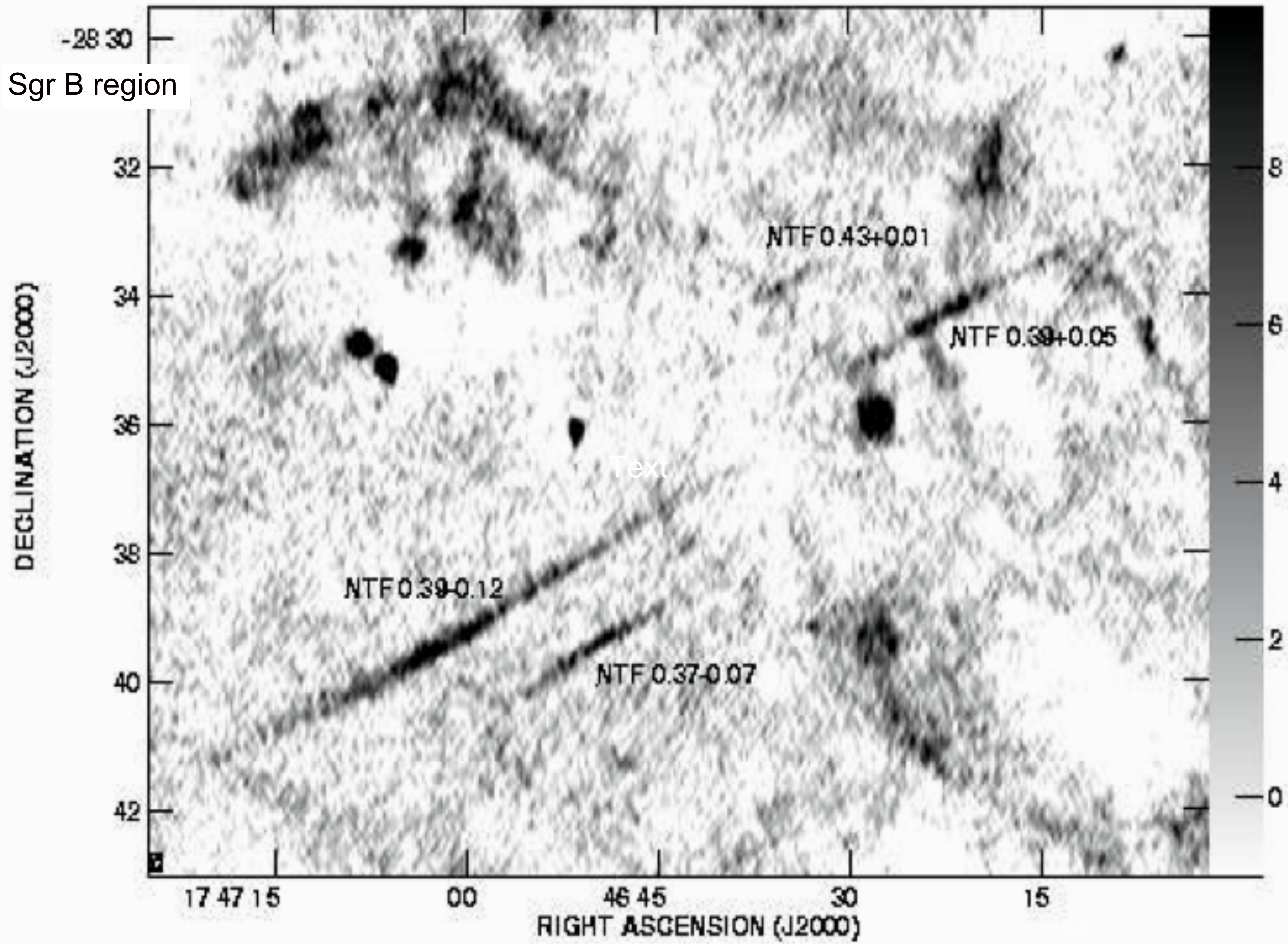
The inner GC region and NTFs at 90 cm radio map; the inner ~1° region, resolution ~10″

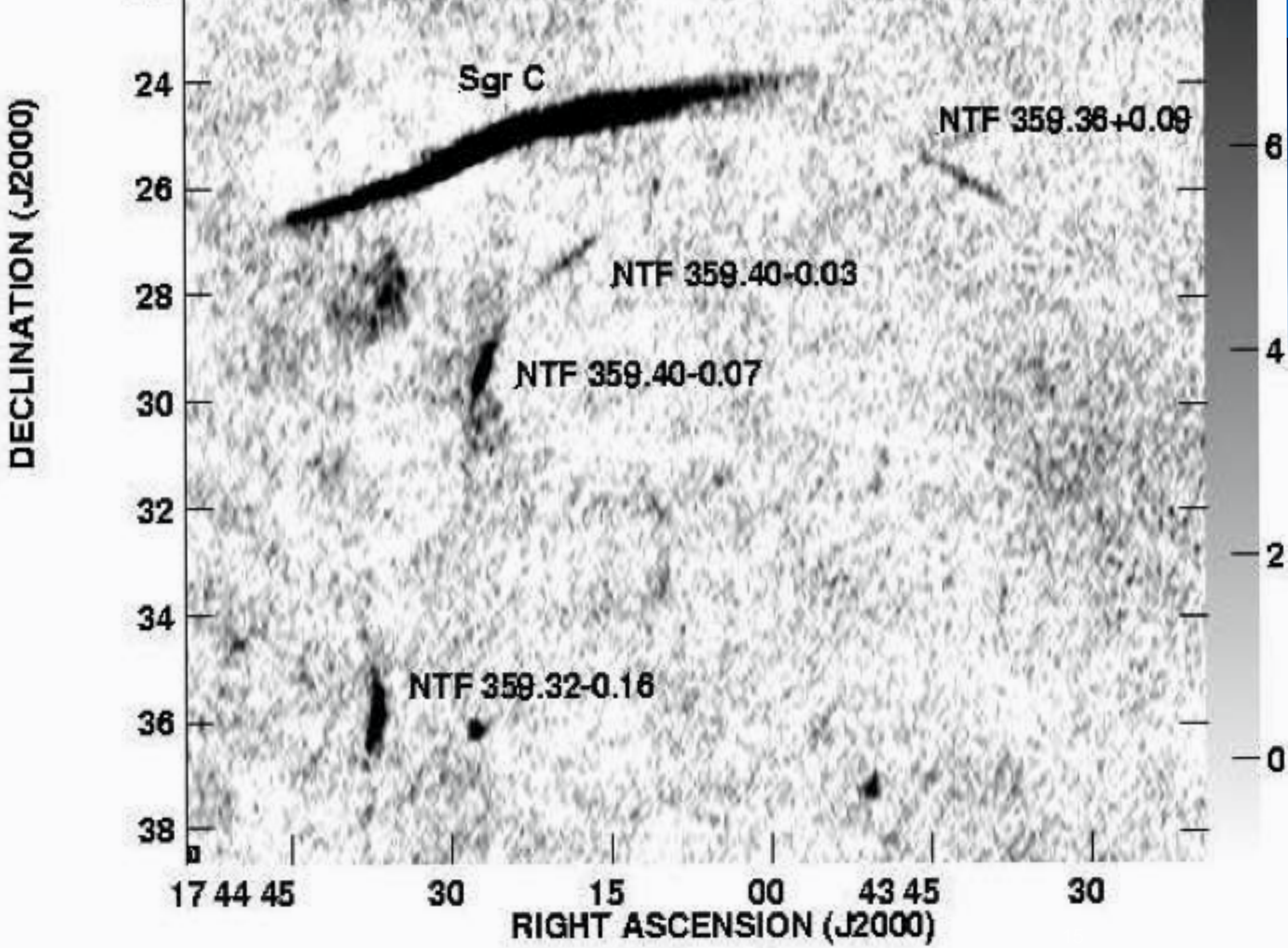
(LaRosa & Nord 2004)


Wide-Field Radio Image of the
Galactic Center
λ = 90 cm
Sgr D HII
Sgr D SNR
SNR 0.9+0.1
Sgr B2
Sgr B1
SNR 0.3+0.0
Threads
New Feature
Arc
Background Galaxy
Threads
Sgr A
Perpendicular thread?
Sgr C
Coherent
structure?
Snake
Mouse
Sgr E
SNR 359.0-00.9
SNR 359.1-00.5
~0.5°
~75 pc
~240 light years
Tornado (SNR?)
Image processing by N.E. Kassim, D.S. Briggs, T.J.W. Lazio, T.N. LaRosa and J. Imamura
Produced at the Naval Research Laboratory, Washington, DC
Original data courtesy of A. Pedlar, K. Anantharamiah, M. Goss, and R. Ekers
(LaRosa et al 2000)


0
100
200
300
-28 42
44
46
48
50
52
DECLINATION (J2000)
(Morris & Yusef-Yadeh 1985)


-28 42
44
46
48
50
52
54
DECLINATION (J2000)
17 45 45
30
15
00
44 45
RIGHT ASCENSION (J2000)

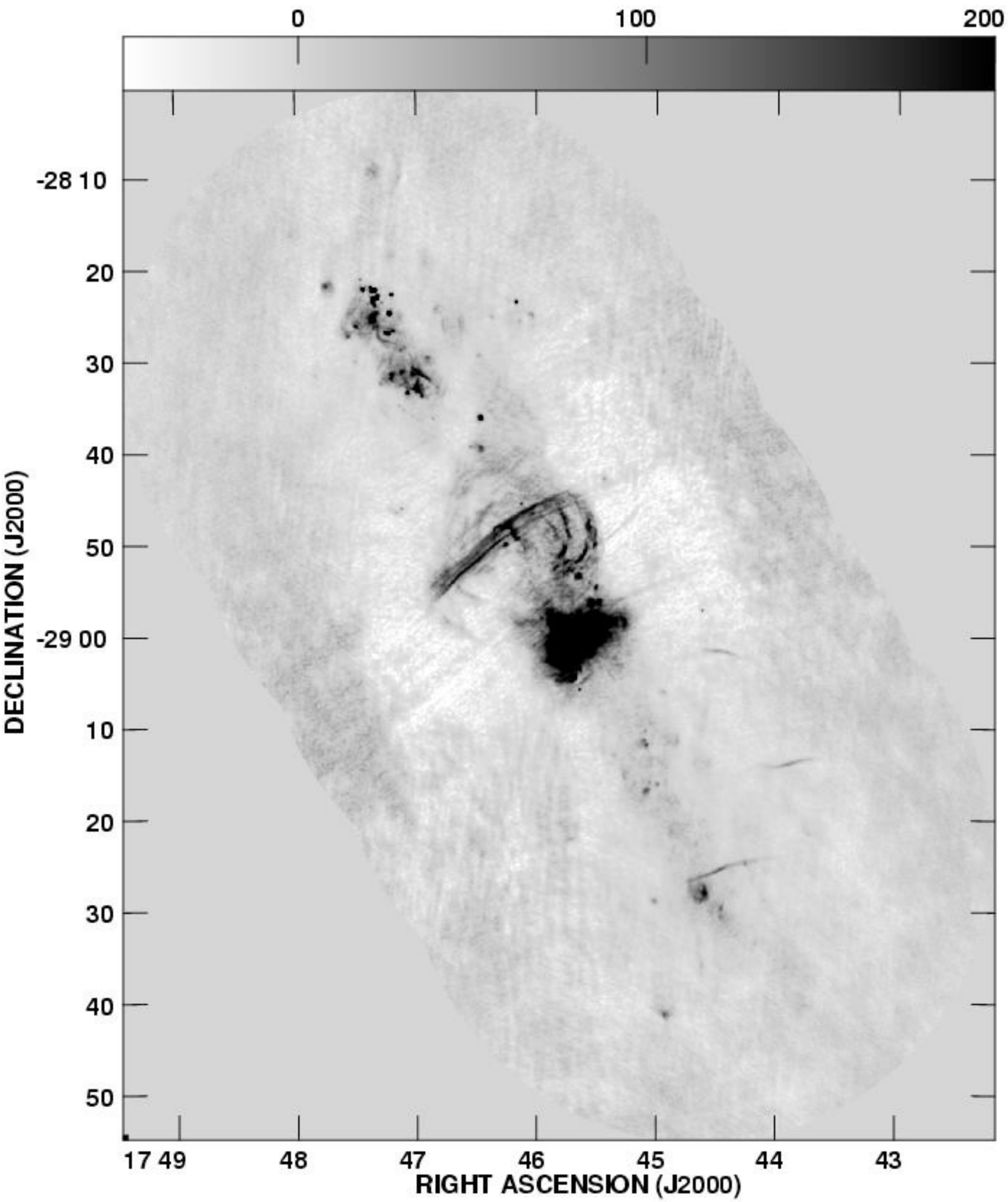

Sgr B region
NTF 0.43+0.01
NTF 0.39+0.05
NTF 0.39-0.12
NTF 0.37-0.07
DECLINATION (J2000)
-28 30
32
34
36
38
40
42
RIGHT ASCENSION (J2000)
17 47 15
00
46 45
30
15
8
6
4
2
0

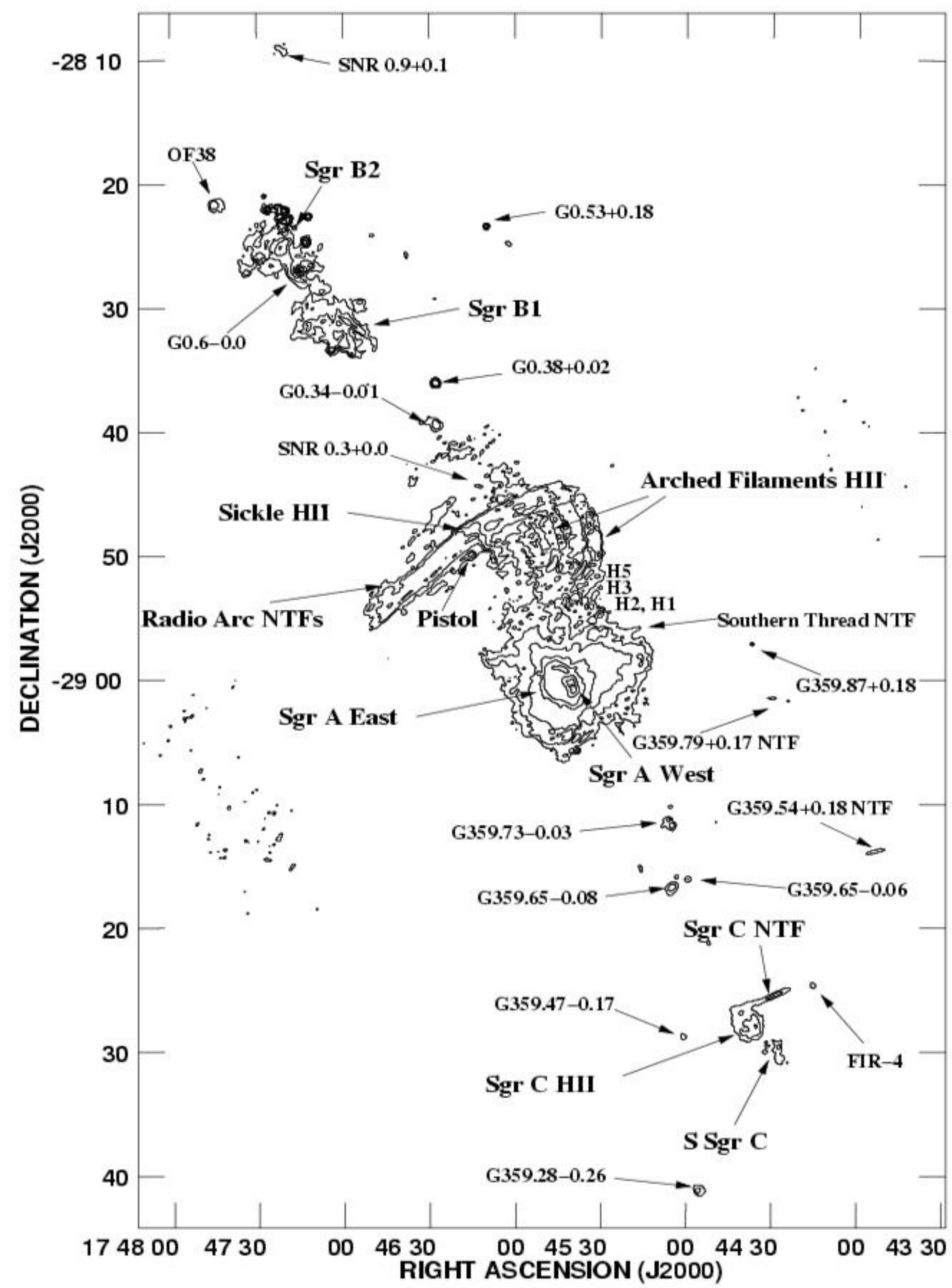


Sgr C
NTF 359.36+0.09
NTF 359.40-0.03
NTF 359.40-0.07
NTF 359.32-0.16
DECLINATION (J2000)
24
26
28
30
32
34
36
38
17 44 45
30
15
00
43 45
30
RIGHT ASCENSION (J2000)
6
4
2
0

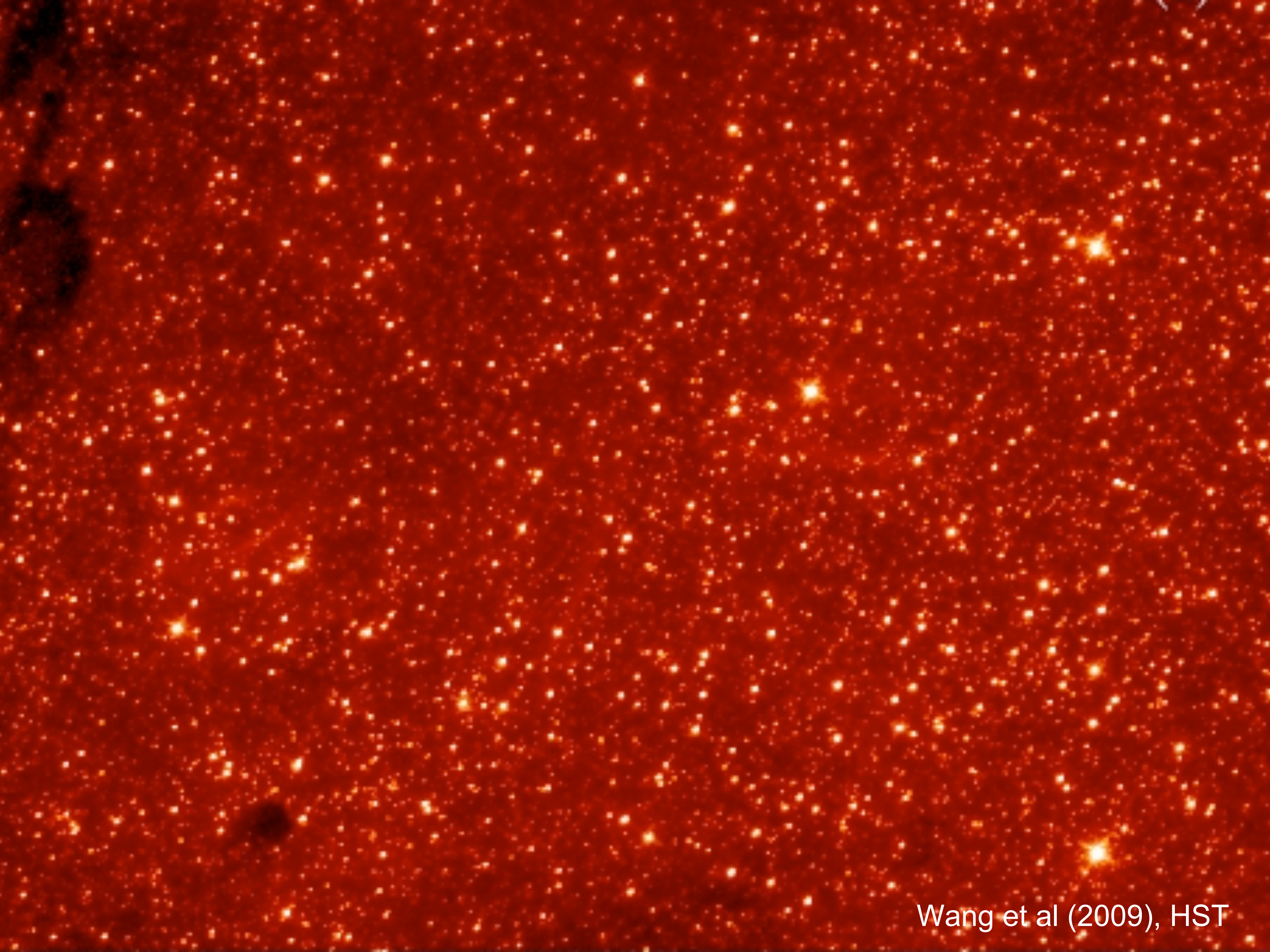
- Filaments remain straight along their length (~1 x 40 pc)
- They cross the Galactic plane almost perpedicularly with little bending, ±20°
- Unique to GC, within ~150 pc from Sgr A*
- Non-thermal spectrum
- Strong linear polarization (~50%)

- Kinked shape can be seen in some filaments ("Snake")
- **B** can reach ~ 1 mG inside the filaments, but only ~10 μG elsewhere in the ISM (?)
- The filaments may be transient (unconfined) structures (?)

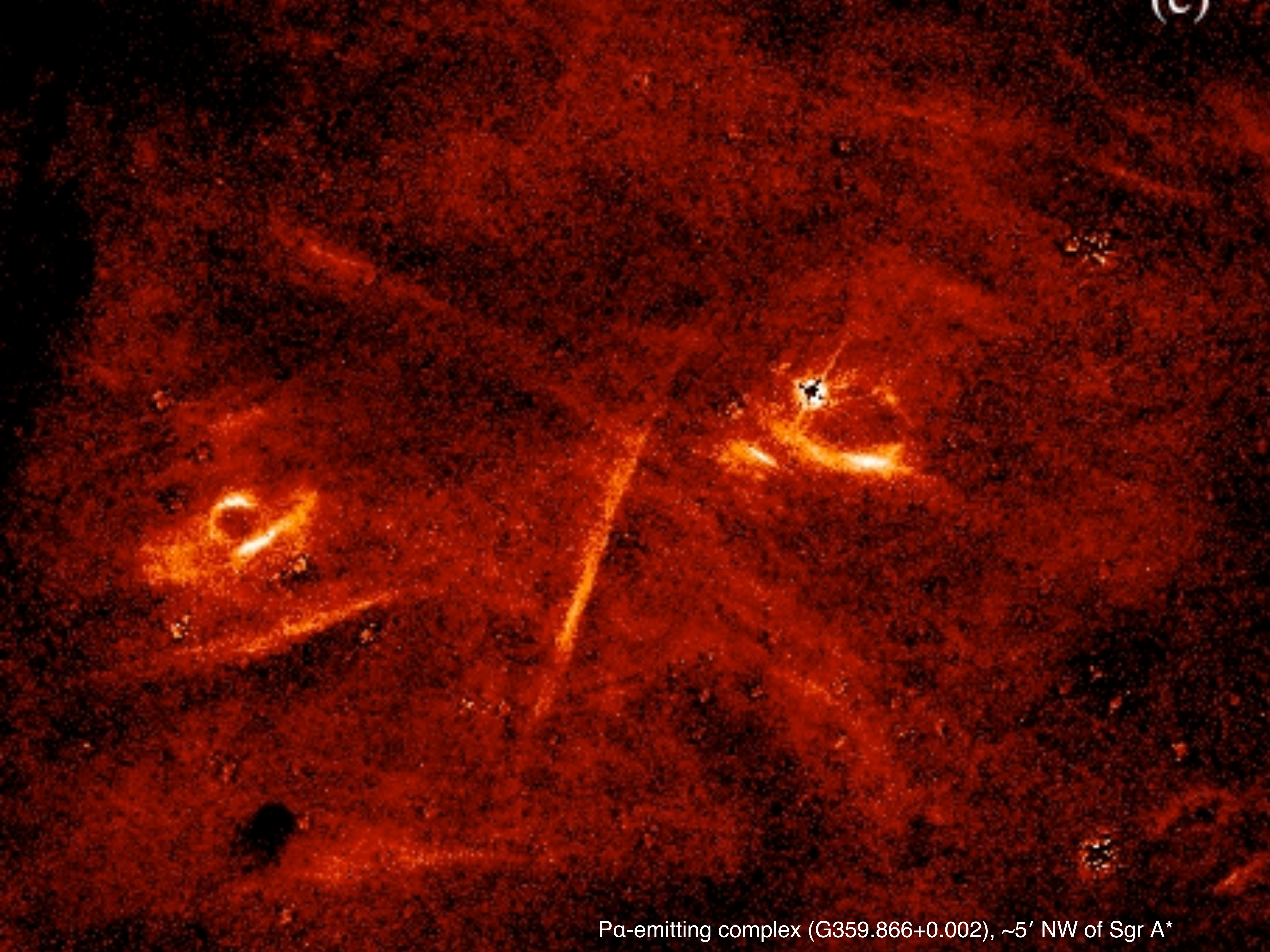




Lang et al. (2010)



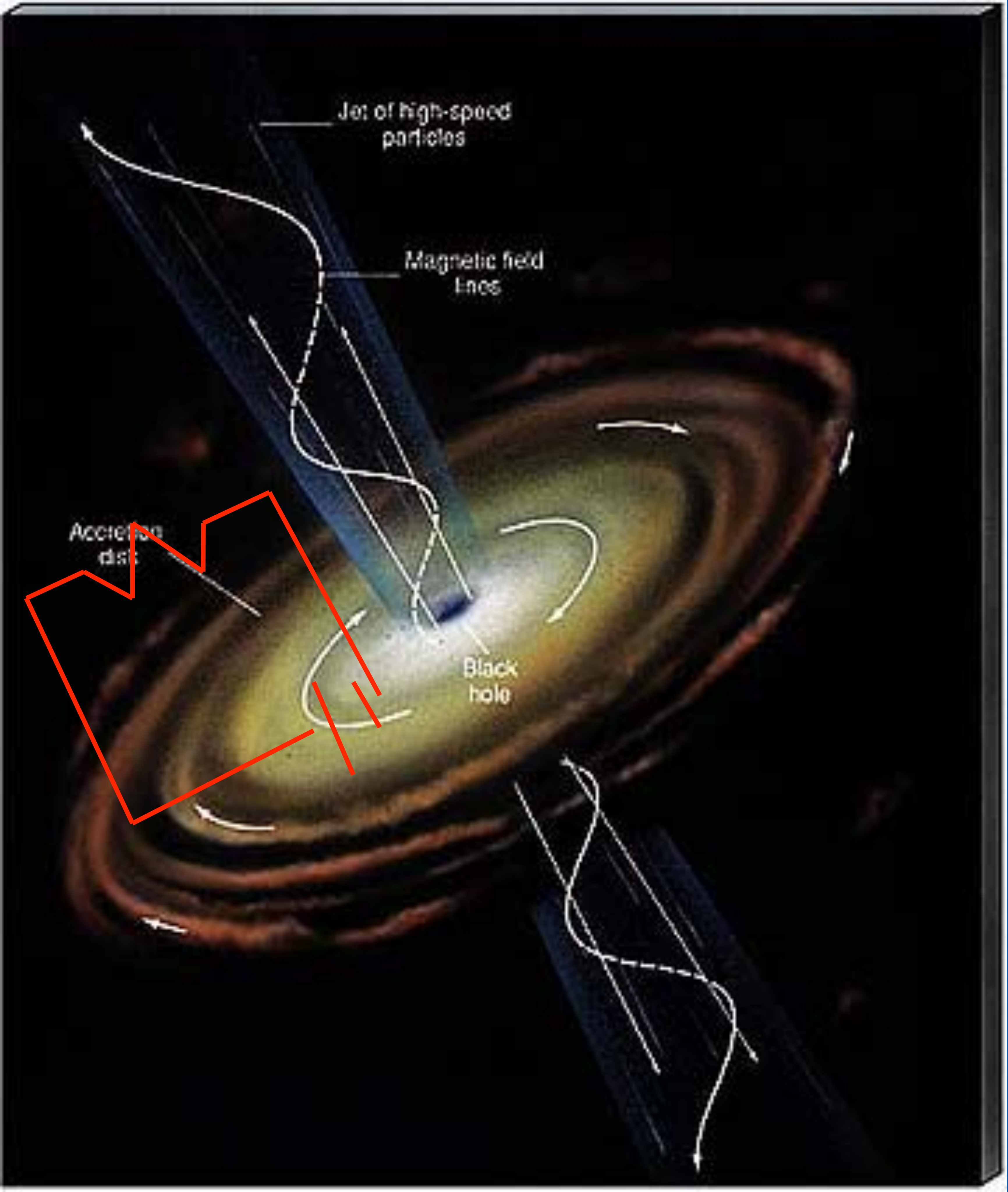
Wang et al (2009), HST



Pα-emitting complex (G359.866+0.002), ~5′ NW of Sgr A*

Motivation #2: Relativistic jets

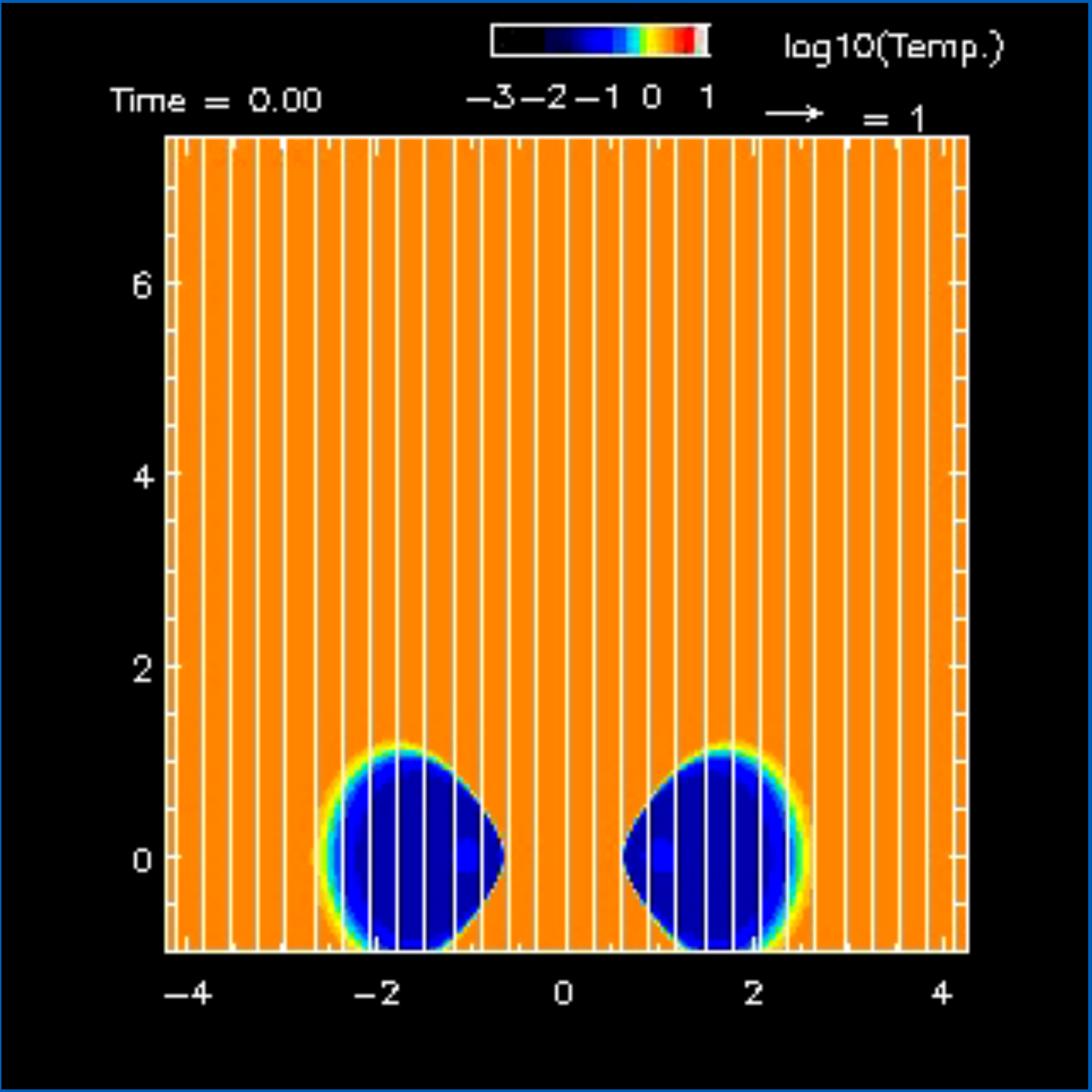
Electromagnetic formation of jets



For a quasar jet:

$B \sim 10^4$ G;
$U \sim 10^{20}$ V
$I \sim 10^{18}$ A
$P \sim 10^{38}$ W

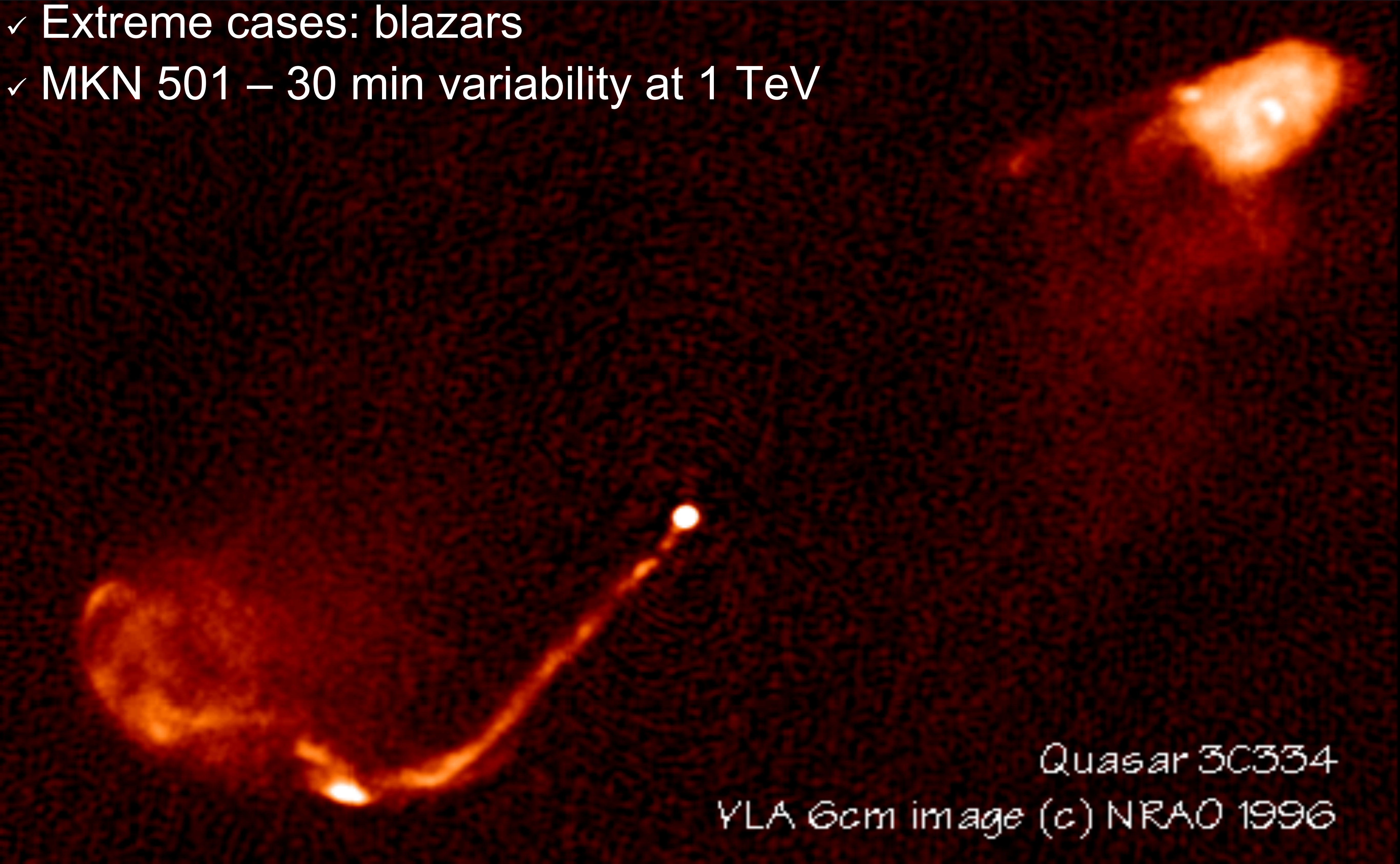
Blandford & Znajek

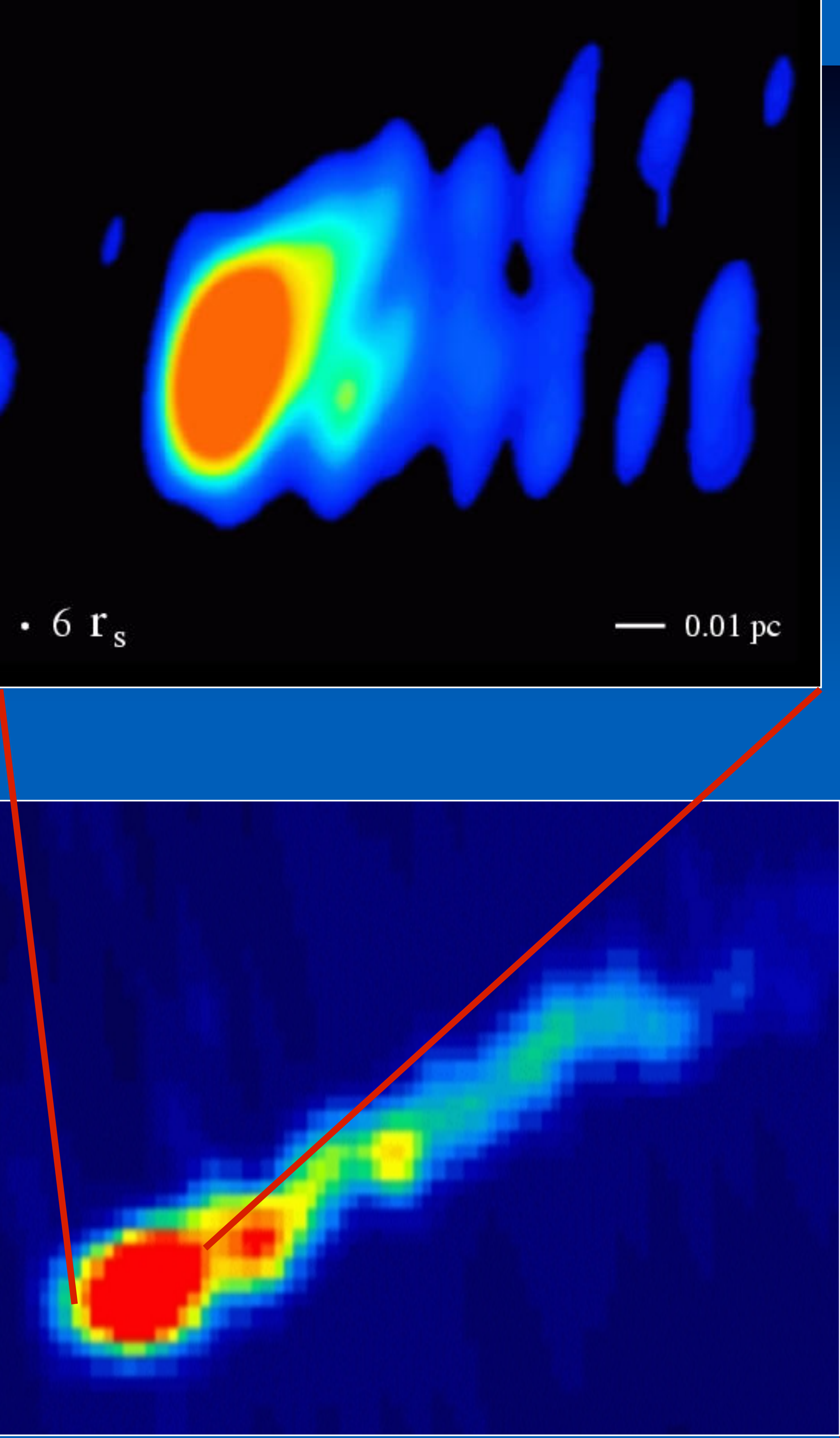


Kudoh

Relativistic jets in quasars

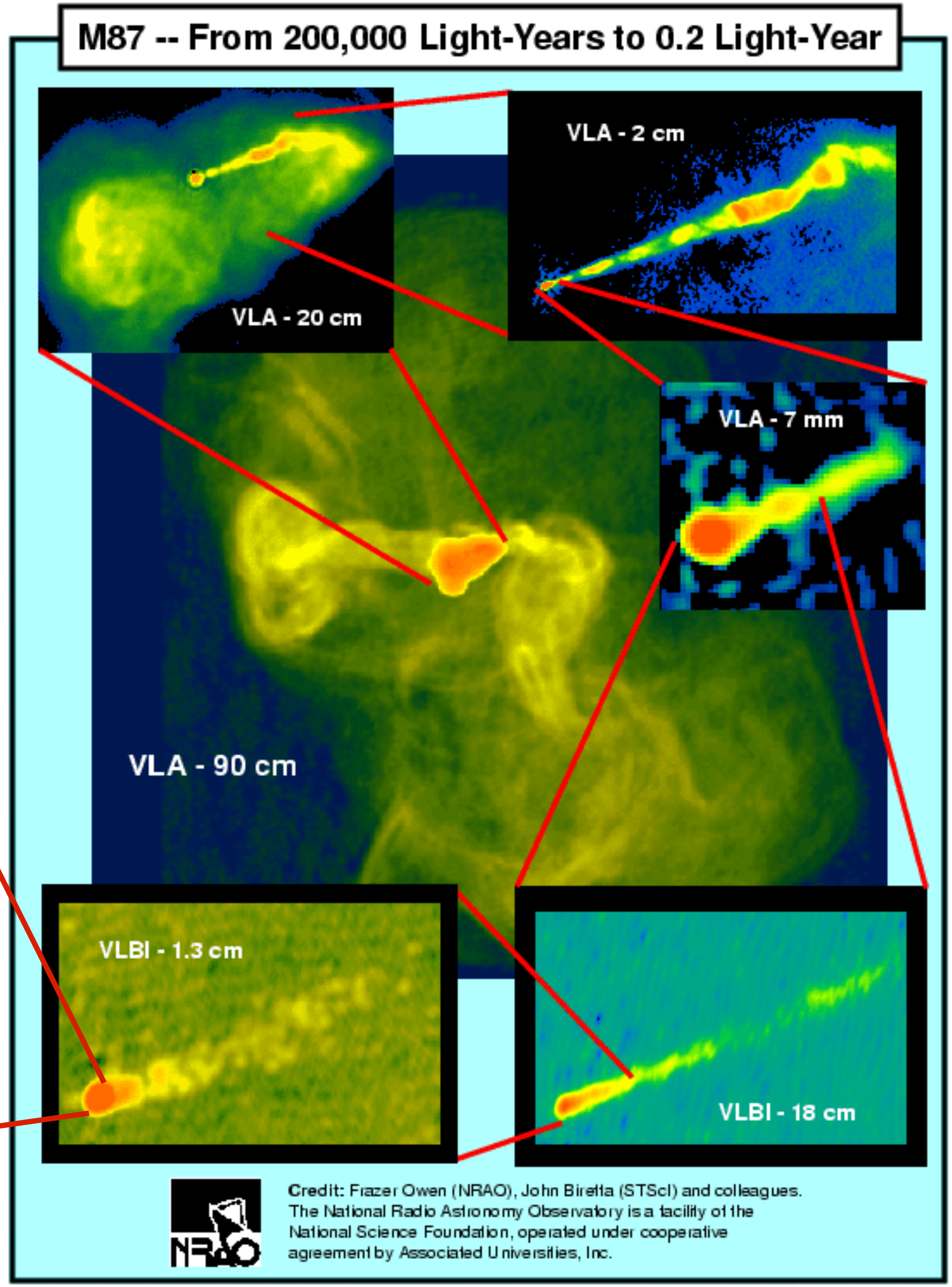
- ✓ Compact radio sources
- ✓ Superluminal motion, v/c ~ 0.99
- ✓ Variable GeV sources
- ✓ Extreme cases: blazars
- ✓ MKN 501 – 30 min variability at 1 TeV





6 r_s
0.01 pc

Halca



M87 -- From 200,000 Light-Years to 0.2 Light-Year
VLA - 20 cm
VLA - 2 cm
VLA - 7 mm
VLA - 90 cm
VLBI - 1.3 cm
VLBI - 18 cm
NRAO
Credit: Frazer Owen (NRAO), John Biretta (STScI) and colleagues.
The National Radio Astronomy Observatory is a facility of the National Science Foundation, operated under cooperative agreement by Associated Universities, Inc.

Curved spacetime

- Space-time metric: $g_{\mu\nu}(r, \theta) = \text{Kerr black hole}$
- Weak, electromagnetic test-fields in a given, fixed background spacetime.

  - Anile A. M., & Choquet-Bruhat Y. (eds.) (1989), *Relativistic Fluid Dynamics*, Lecture Notes in Mathematics 1385 (Springer, Berlin)
  - Lichnerowitz A. (1967), *Relativistic Hydrodynamics and Magnetohydrodynamics* (Benjamin, New York)
  - Znajek R. M. (1976), *Black Hole Electrodynamics*, PhD Dissertation (Univ. of Cambridge)
  - Phinney E. S. (1983), *A Theory of Radio Sources*, PhD Dissertation (Univ. of Cambridge)

$c = G = 1$, and the signature of metric $-$+++

Conservation laws

- Conservation of the particle number:

$$(\rho_0 u^\alpha)_{;\alpha} = 0, \qquad \rho_0 = mn,$$

m is the particle rest mass,
n numerical density,
u^α four-velocity.

Note: we do not consider a possibility of pair creation.

- Normalization condition for four-velocity:

$$u^\alpha u_\alpha = -1$$

- Energy-momentum conservation:

$$T^{\alpha\beta}{}_{;\beta} = 0.$$

Energy-momentum tensor

$$T^{\alpha\beta} = T^{\alpha\beta}_{\rm matter} + T^{\alpha\beta}_{\rm EMG}$$

with

$$\begin{aligned} T^{\alpha\beta}_{\rm matter} &= (\rho + p) u^\alpha u^\beta + p g^{\alpha\beta}, \\ T^{\alpha\beta}_{\rm EMG} &= \frac{1}{4\pi}\left(F^{\alpha\mu} F^{\beta}_{\mu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} g^{\alpha\beta} \right). \end{aligned}$$

Electromagnetic field tensor:

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}.$$

Force-free condition

$$F_{\alpha\beta}u^{\beta} = 0.$$

This can be written in the form

$$F_{\alpha\beta}u^{\beta} = \Gamma \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} \begin{pmatrix} -1 \\ v_x \\ v_y \\ v_z \end{pmatrix} = 0.$$

Notice: ideal MHD approximation reads (non-relativistic form)

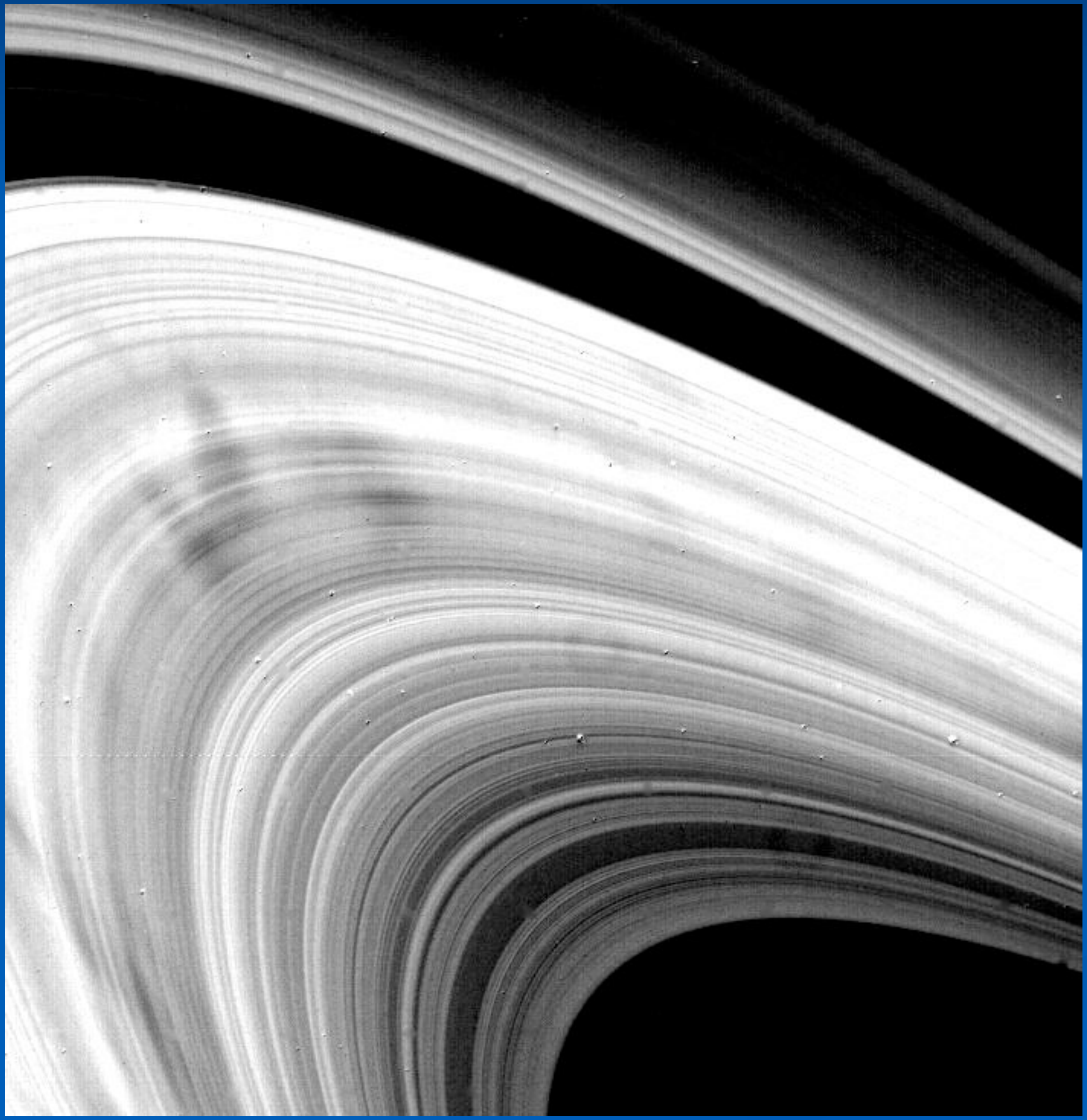
$$\boldsymbol{E'} \equiv \boldsymbol{E} + c^{-1}\boldsymbol{v} \times \boldsymbol{B} = 0.$$

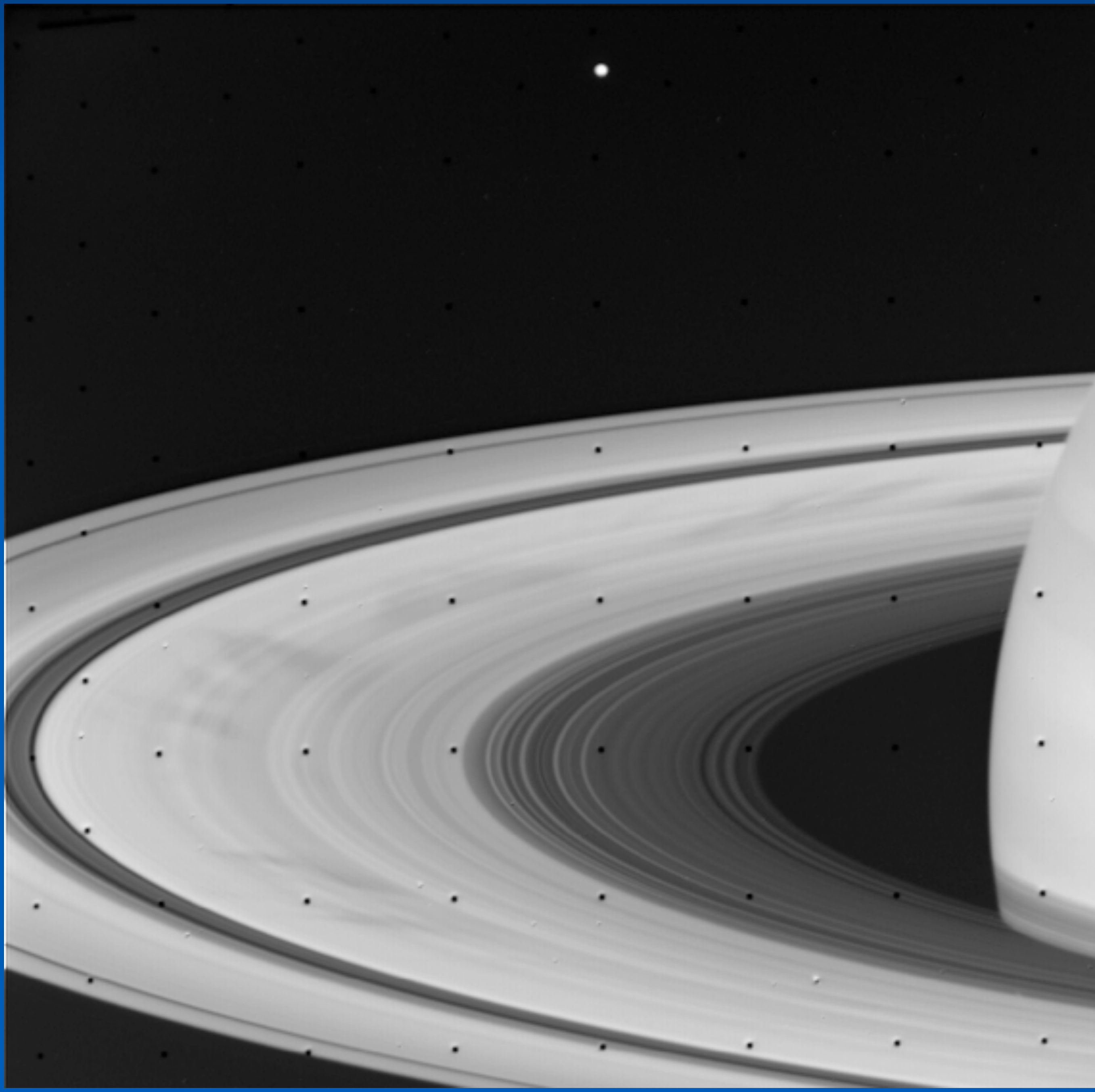
Force free approximation reads

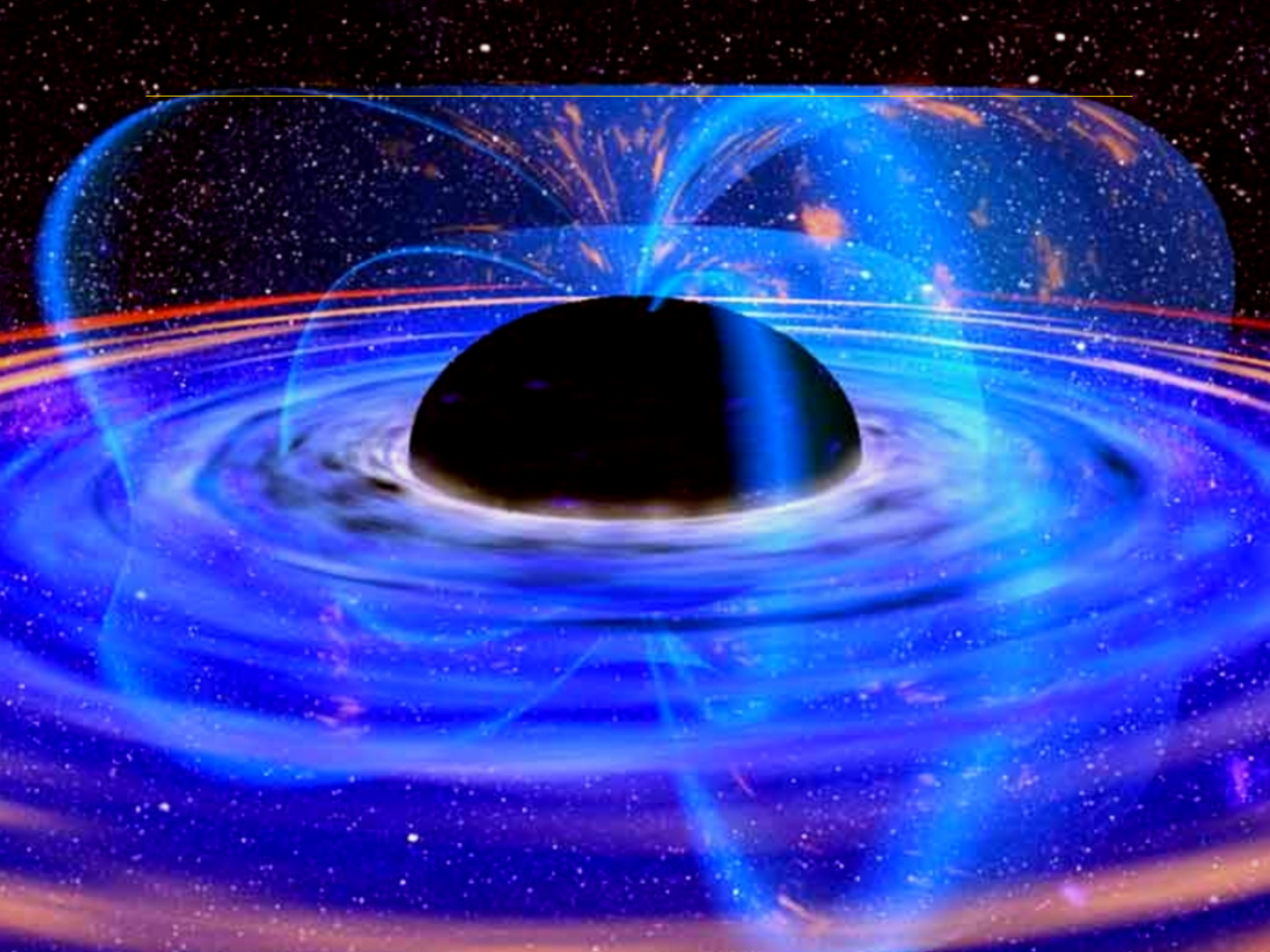
$$\rho_{\mathrm{e}}\boldsymbol{E} + c^{-1}\boldsymbol{j} \times \boldsymbol{B} = 0.$$

Motivation #3: Spokes in Saturn's B-ring

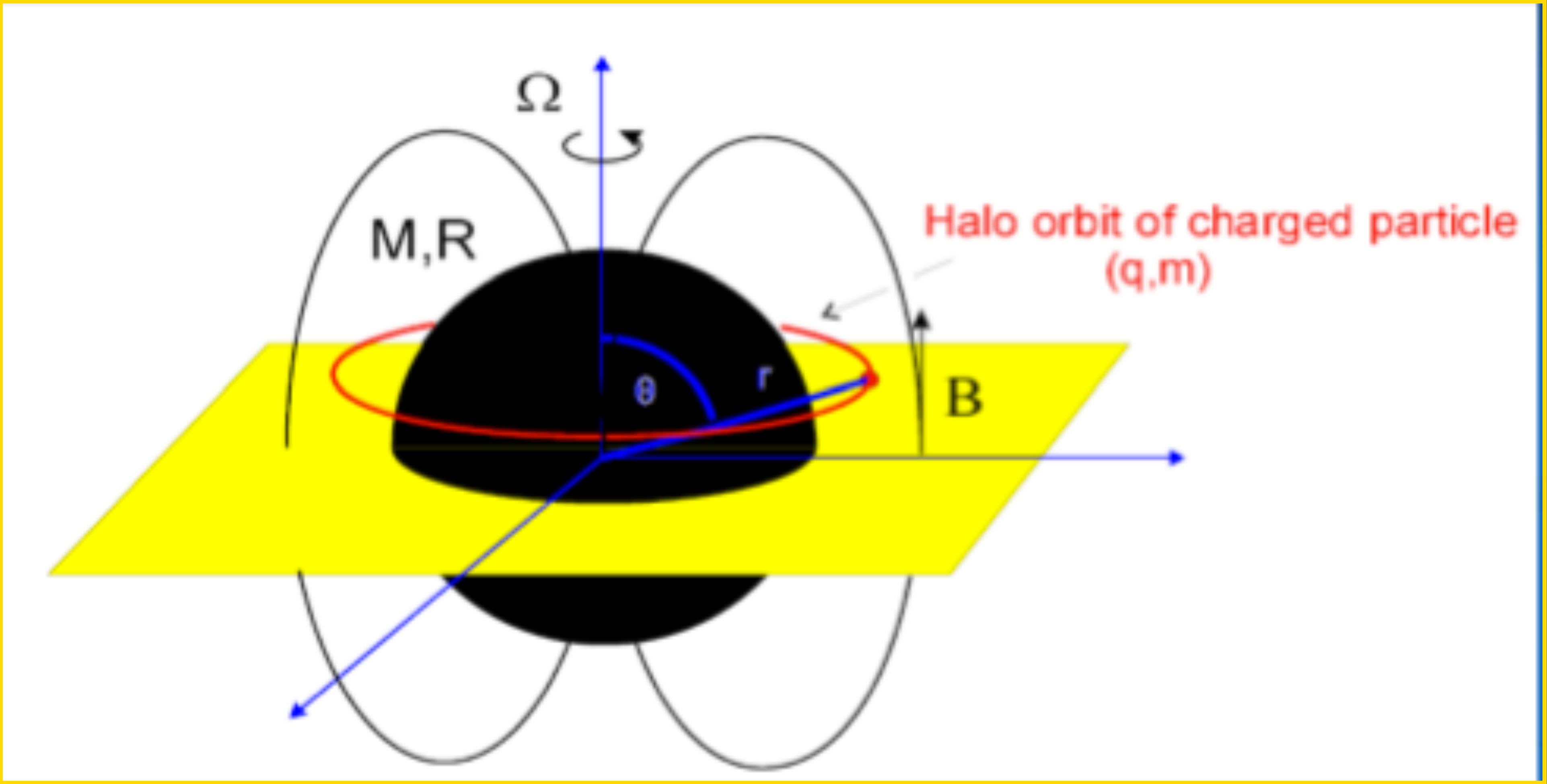
(from Voyager 2)





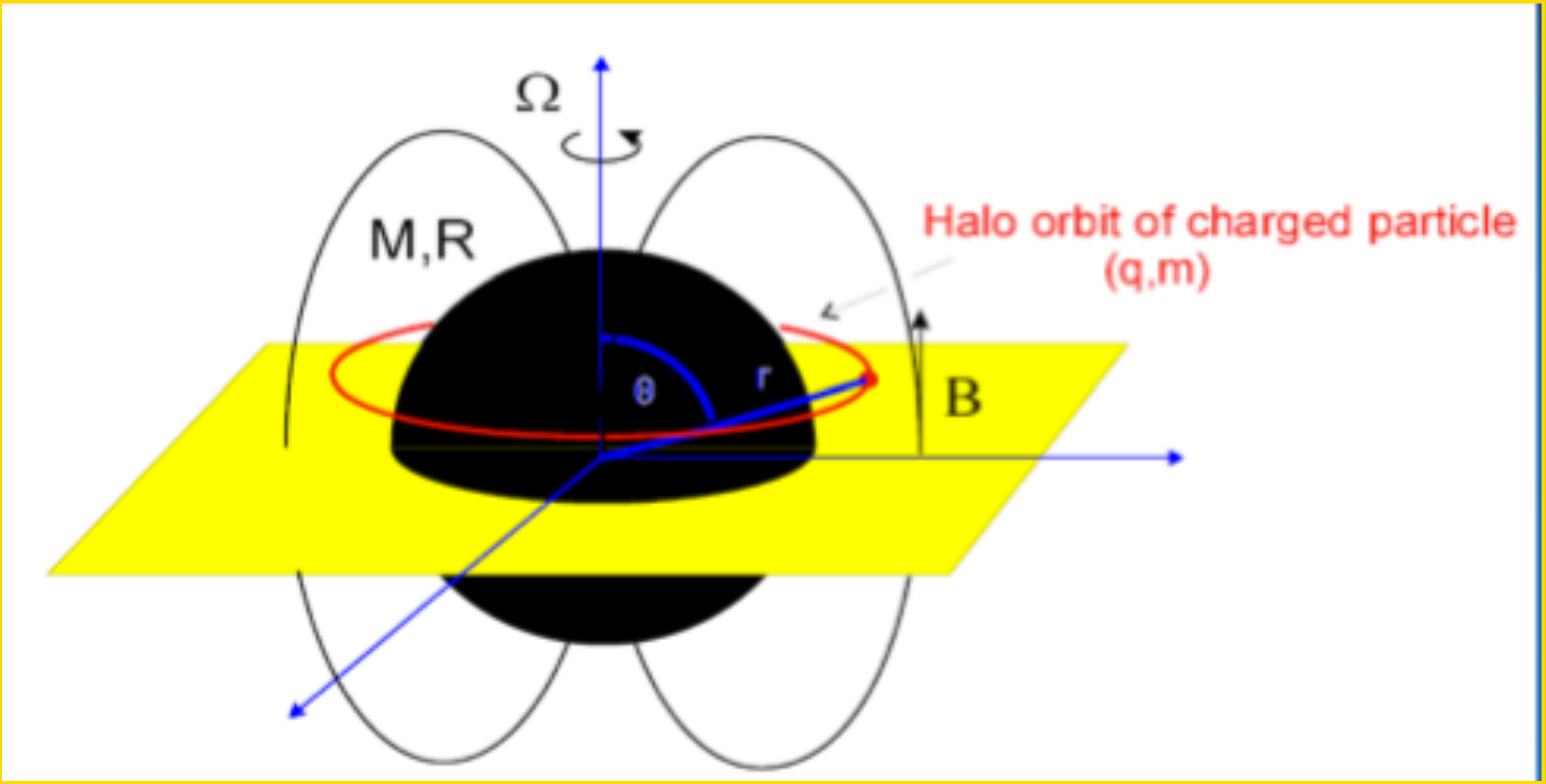


Context of particle orbits



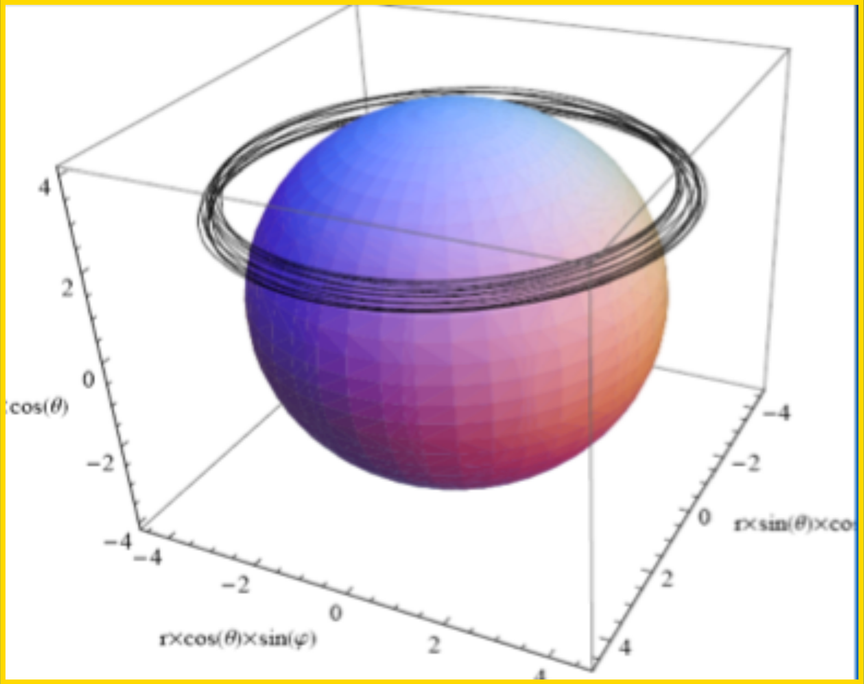
Context of particle orbits

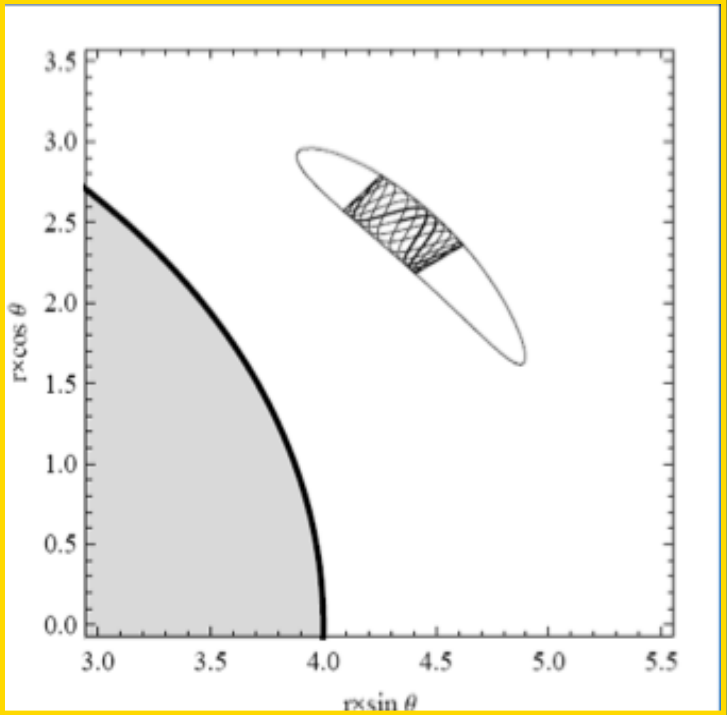
- 'Halo orbits' – off-equatorial circular orbits at constant r and θ

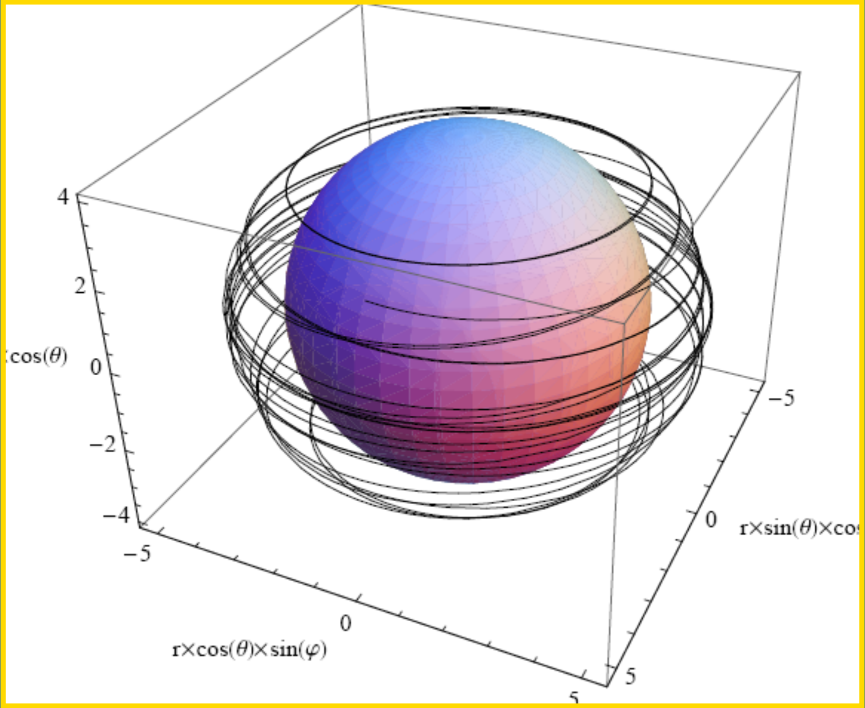


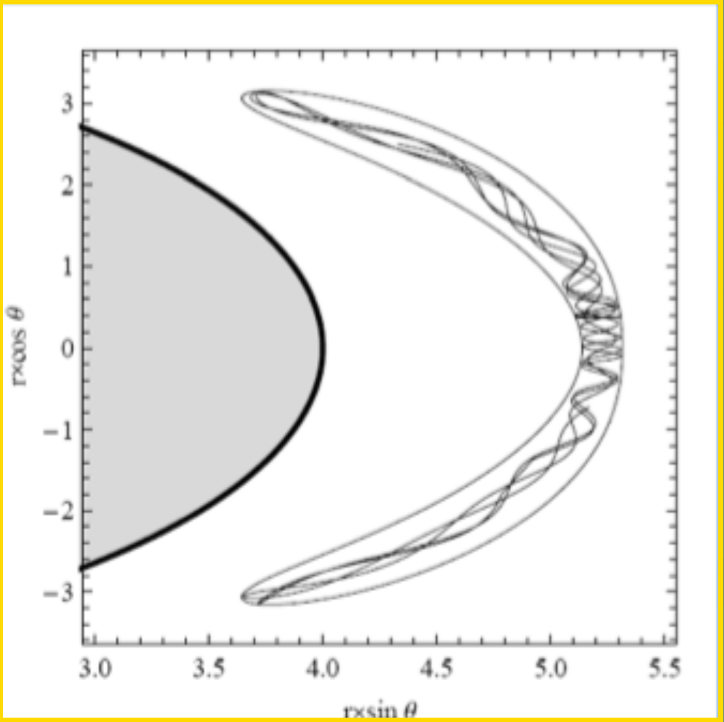
Schwarzschild geometry + rotating dipole MF	slowly rotating neutron star
Kerr geometry + uniform magnetic field	Kerr BH embedded in MF
Kerr-Newman geometry	Kerr-Newman BH

Effective potential and examples of trajectories









(Kovář et al., CQG, 2010)

Dusty plasmas

Coulomb vs gravitational forces on dust grains,

$$G\,\frac{m_1\,m_2}{d^2} = \frac{q_1\,q_2}{4\pi\epsilon_0 d^2}\;e^{-\frac{d}{\lambda_{\rm D}}}$$

$$m_i \simeq \tfrac{4}{3}\pi a^3 \rho, \qquad q_i \simeq 4\pi\epsilon_0 a\phi$$

(Horányi 1996; Ishihara 2007)

Dusty plasmas

$$a \simeq \left(\frac{9\epsilon_0}{4\pi G}\right)^{\frac{1}{4}} \left(\frac{\phi}{\rho}\right)^{\frac{1}{2}} \simeq 0.6\sqrt{\frac{\phi}{\rho}}$$

For $\phi \simeq 1\mathrm{V}$, $\rho \simeq 10^4 \mathrm{kg\,m^{-3}}$
$\rightarrow a \simeq 6\mathrm{mm}.$

In a case where the electron and ion thermal currents are the only charging currents, the equilibrium potential of a grain in a plasma is

$$\phi = -\beta kT/e.$$

In 1 eV hydrogen plasma $\phi = -2.5$ V. a~1 μm radius particle will collect ~1800 extra electrons. Charges then fluctuate around their equilibrium value.

Dusty plasmas

Charging of dust grains,

$$\frac{dQ}{dt} = \sum_{\alpha} I_{\alpha} \simeq 0,$$

$$I_{\alpha} = I_{\mathrm{i}},\, I_{\mathrm{e}},\, I_{\mathrm{ph}} \ldots$$

$$I_{\alpha} = 4\pi a^2 \int_{v_{\alpha}^{*}}^{\infty} \int_{0}^{\frac{\pi}{2}} \int_{0}^{2\pi} v \, \cos\theta \, f_{\alpha}(v) \, v^2 \, d\Omega \, dv$$

Dusty plasmas

$$I_\mathrm{e} = -\tfrac{1}{4} e\, n_\mathrm{e}\, v_\mathrm{e}\, S\, \mathrm{e}^{-z},$$

$$v_\mathrm{e} = \sqrt{\frac{4}{\pi}\frac{2kT_\mathrm{e}}{m_\mathrm{e}}}, \qquad S = 4\pi a^2,$$

$$z = -\frac{e\phi}{kT_\mathrm{e}} = \ln\left(1 + \frac{a\,\omega_\mathrm{p}\,t}{\sqrt{2\pi}\lambda_\mathrm{D}}\right)$$

$$\omega^2 = n_\mathrm{e} e^2/\epsilon_0 m_\mathrm{e},\ \lambda_\mathrm{D}^2 = \epsilon_0 k T_\mathrm{e}/n_\mathrm{e} e^2.$$

Equations of GRMHD

$$\boxed{T^{\alpha\beta}{}_{;\beta} = 0}$$

$$\boxed{F^{\alpha\beta}{}_{;\beta} = -J^{\alpha}}$$

$$J^{\alpha} = \epsilon U^{\alpha} + \sigma F^{\alpha\beta} U_{\beta}$$

$$T^{\alpha\beta} = T^{\alpha\beta}_{\rm MAT} + T^{\alpha\beta}_{\rm EM}$$

$$T^{\alpha\beta}_{\rm MAT} = (\rho + p) U^{\alpha} U^{\beta} + p g^{\alpha\beta}$$

$$T^{\alpha\beta}_{\rm EM} = \frac{1}{4\pi}\left(F^{\alpha}_{\gamma} F^{\beta\gamma} - \frac{1}{4} F_{\gamma\delta} F^{\gamma\delta} g^{\alpha\beta}\right)$$

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$$

- Toy model: test non-conductive charged perfect fluid
  - axial symmetry
  - polytropic equation of state

$$\boxed{\Rightarrow}$$

$$\boxed{T^{\alpha\beta}{}_{;\beta} = -F^{\alpha\beta} J_{\beta}}$$

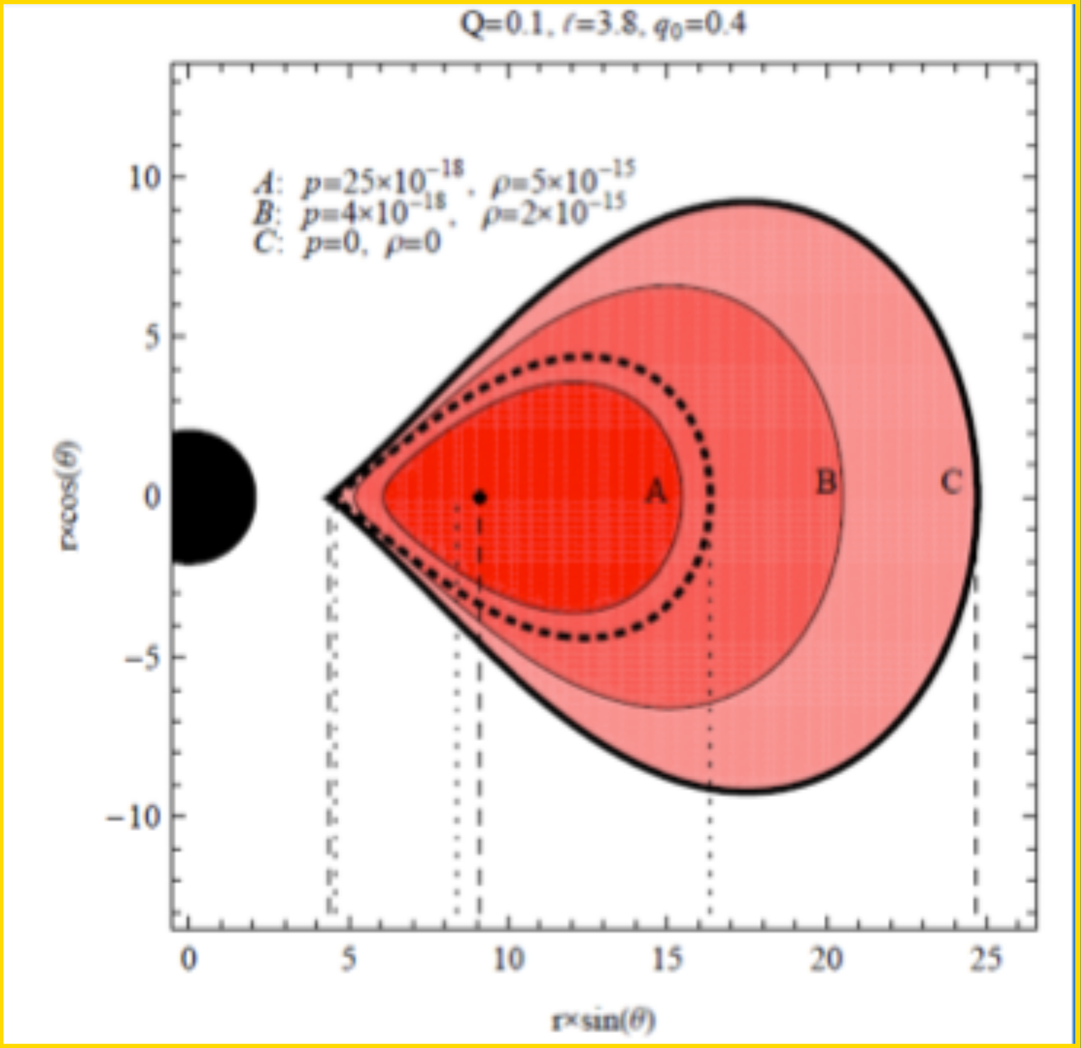
$$p = K\rho^{\Gamma}$$

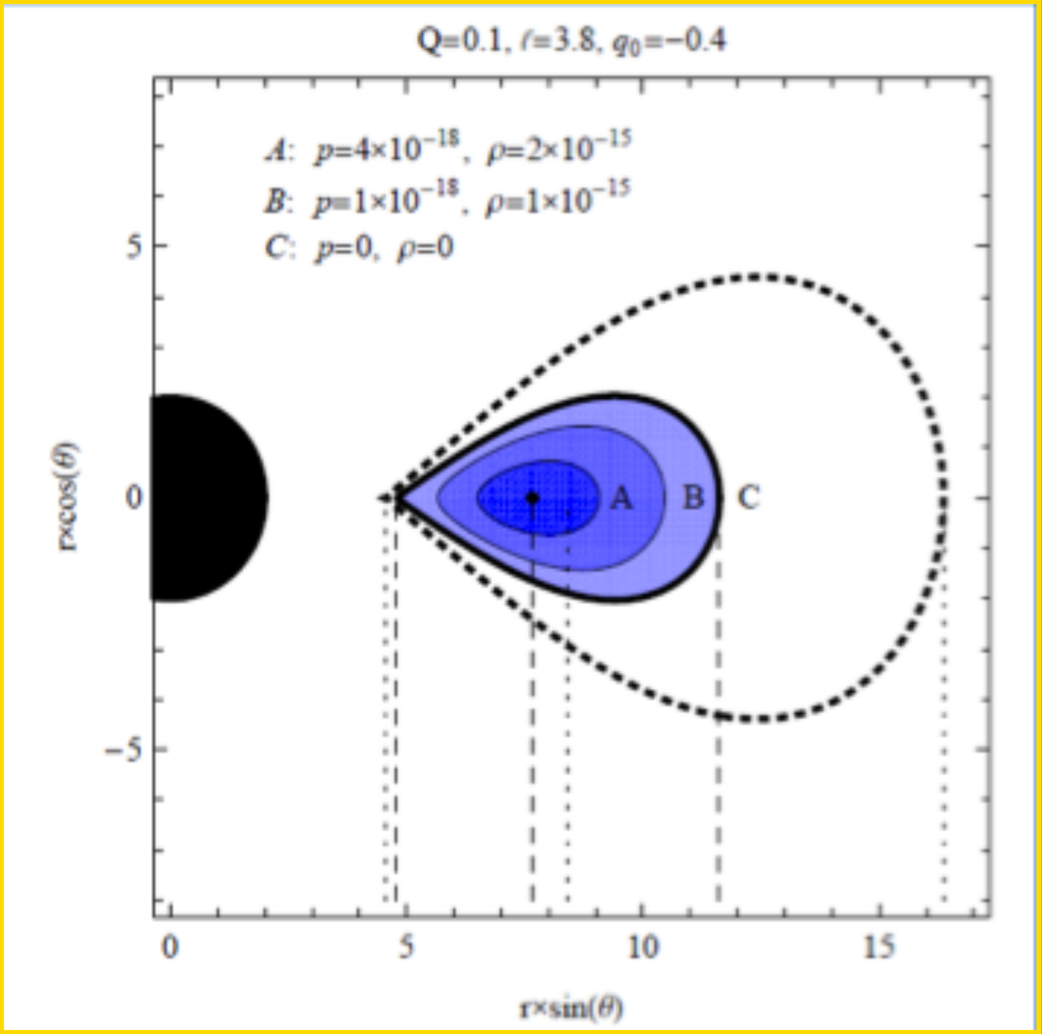
$$U^{\alpha} = (U^{t}, U^{\phi}, 0, 0)$$

$$J^{\alpha} = \epsilon U^{\alpha}$$

Kovář et al. (2011)

Pressure and density profiles (Reissner–Nordström black hole)





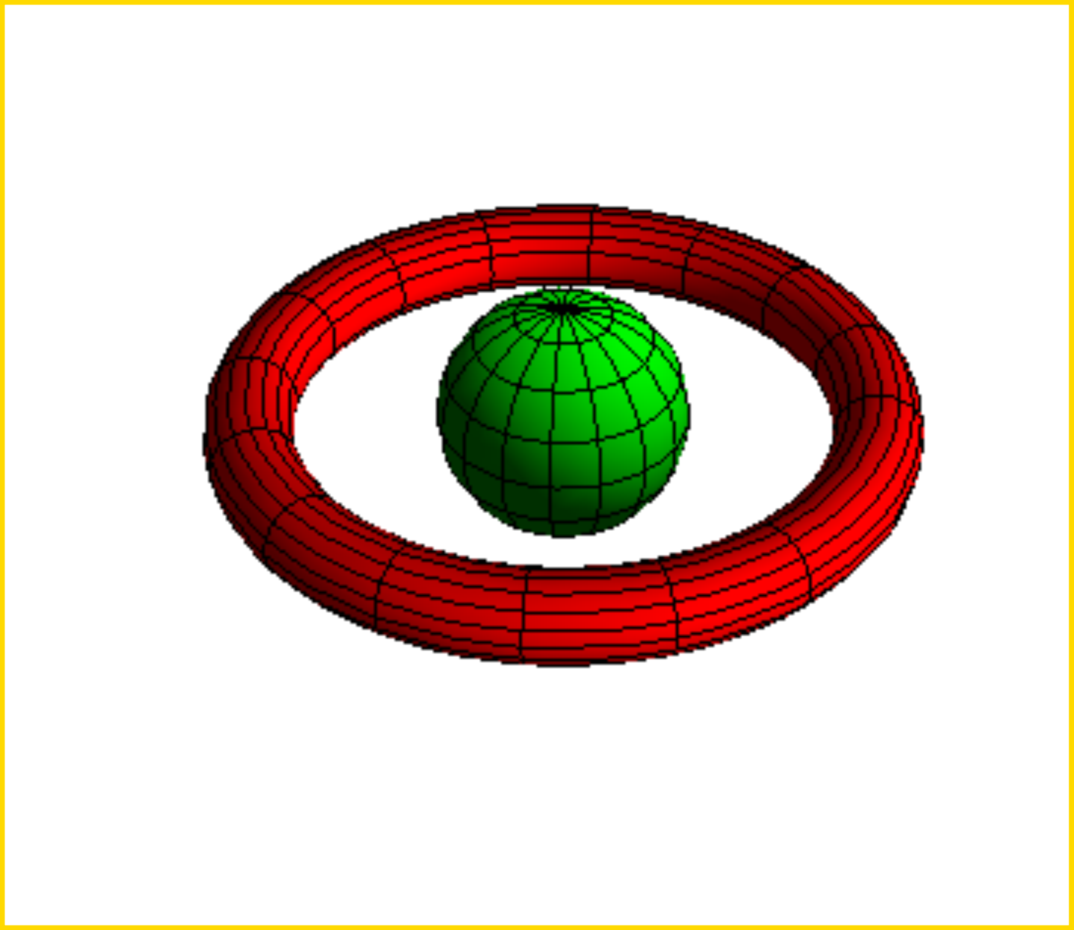
Positively charged

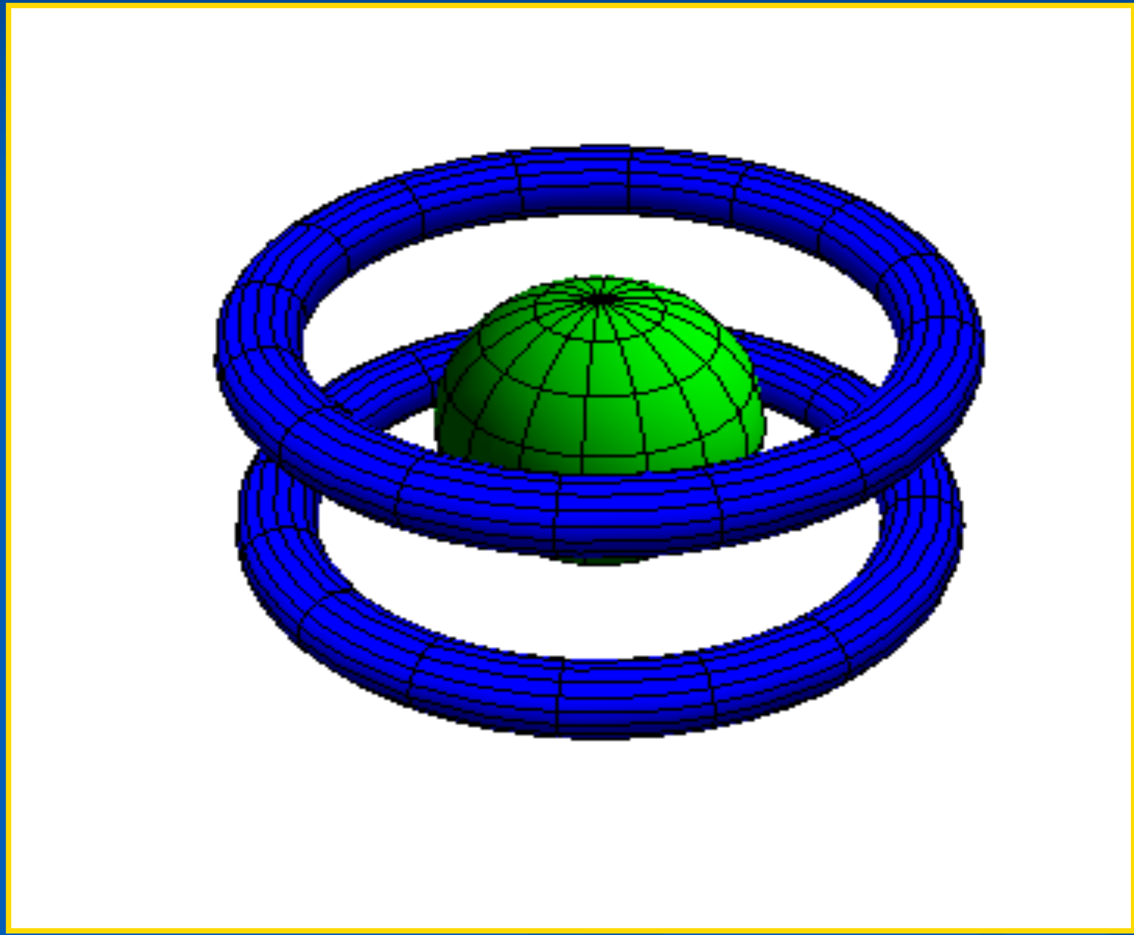
Negatively charged

Hydrodynamical tori

and

electrically charged tori





Summary and References

1. We construct a charged thick torus model around black-hole in GR.
2. We also construct these tori in Newtonian limit with a magnetic dipole.
3. We find off-equatorial structures emerging above the equatorial plane.

[1] Karas, Kopáček, & Kunneriath (2012),
Classical and Quantum Gravity, 29, id. 03501

[2] Kovář, Slaný, Cremaschini, Stuchlík, Karas, & Trova (2014)
Physical Review D, 90, 044029

[3] Kovář, Slaný, Stuchlík, Karas, Cremaschini, & Miller (2011)
Physical Review D, 84, 084002

[4] Slaný, Kovář, Stuchlík, & Karas (2013)
Astrophysical Journal, 205, id. 3